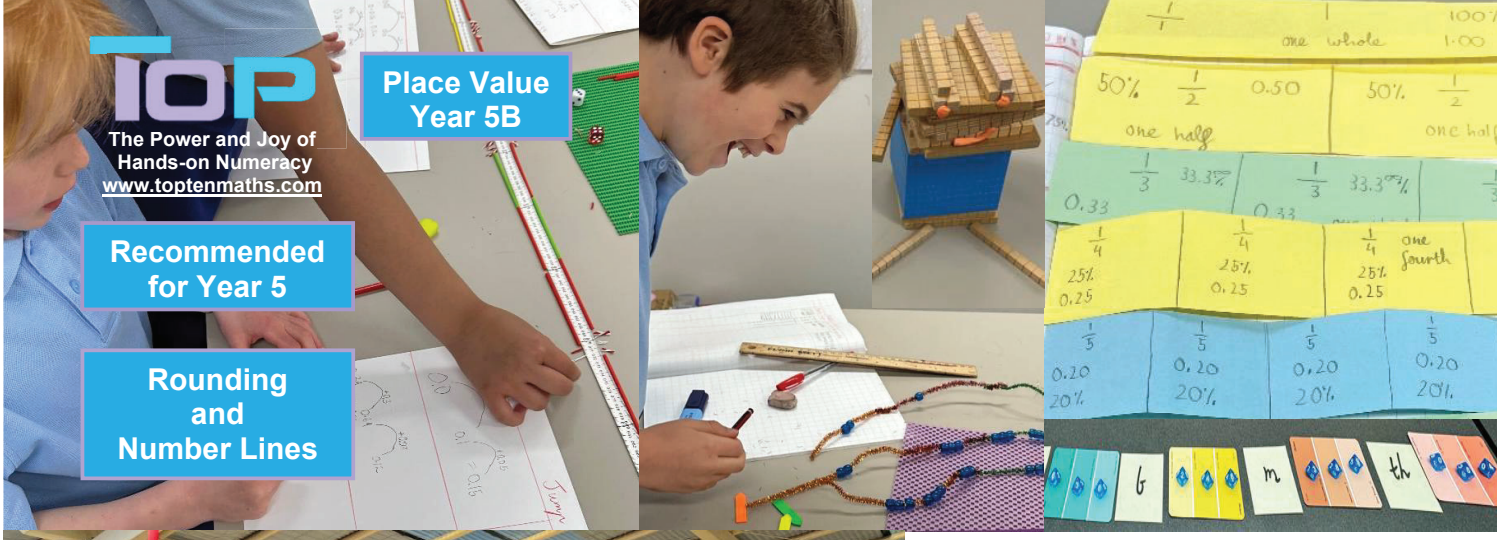


Real-Life Numeracy Years 3-6

Planning Package

Sequential units with hands-on, real-life numeracy for Year 3, Year 4, Year 5 and Year 6 students

Ten years of development in Australian classrooms.

Genuinely high engagement and conceptual understanding in middle to upper primary numeracy.

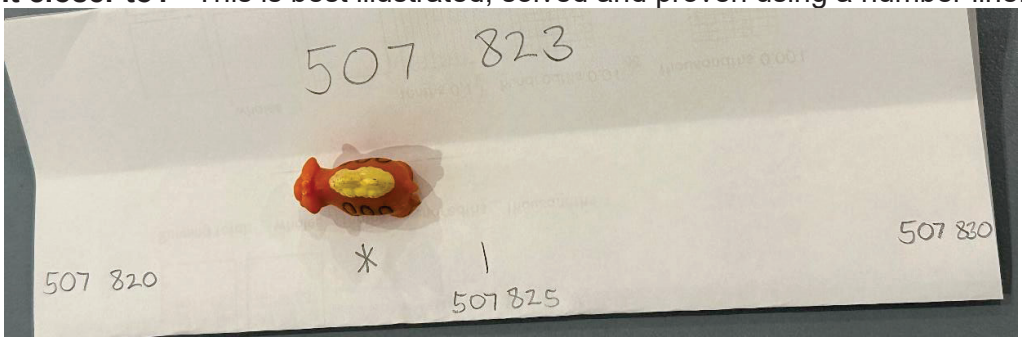
Comprehensive differentiation for wide ranges: Pre-planned and workable enabling and extending prompts for every lesson.

High-impact, high-relevance professional learning on a daily basis to support planning.

Comprehensive diagnostic and formative assessments to target each sequential point-of-need.

Teaching Tips – Rounding

Avoid ‘rounding rollercoasters’ or ‘rounding mountains’ and instead repeat this critical question: **“What is it closer to?”** This is best illustrated, solved and proven using a number line.



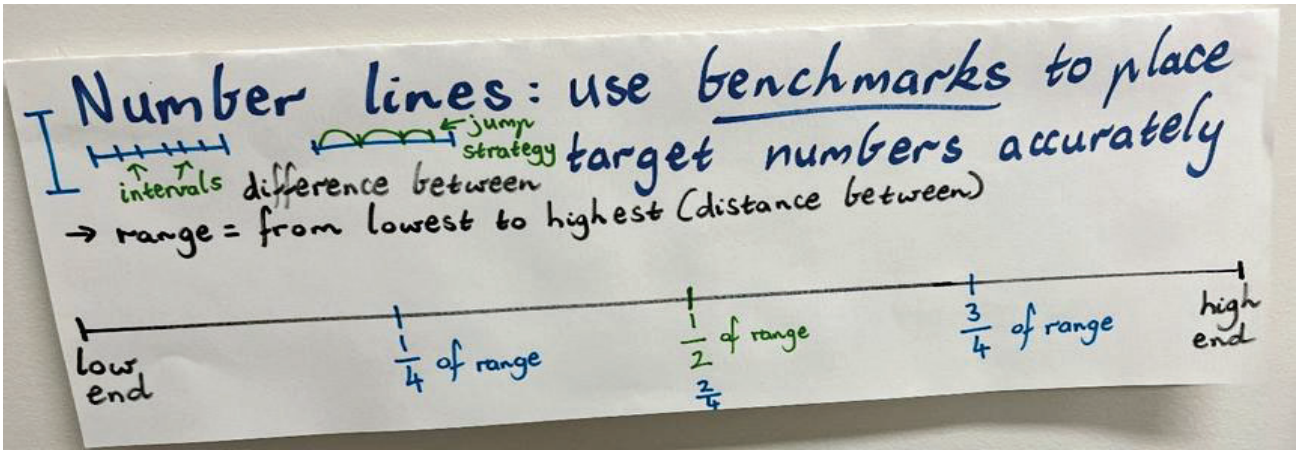
Rote rules relating to underlining the place next door, and so on, often falter in students’ memories when they are conceptually challenged, and also do not create genuine number sense in terms of number line awareness. These fragile procedures frequently fall to pieces when students are asked to round inside a place value (rounding a tens of thousands number to the nearest ten), or estimate while operating, as the rules are too fragile to be applied meaningfully and with a number sense that is critical for real-life numeracy.

Research quoted by Clarke et al (2008): **According to this large-scale study, if we do not teach or emphasise rounding and estimation throughout the year of numeracy, we are setting students up to fail in more than 60 percent of real-life scenarios.**

> Surveys completed by two hundred adults over a twenty-four-hour period found that more than 60 percent of all calculations carried out in daily life only required an estimate (Northcote and McIntosh 1999). We believe that the curriculum emphasis should reflect this finding. This is one reason why teaching fraction algorithms for the four operations does not prepare students for real-life encounters with fractions, where mental estimation is the key skill.

The discussion point then becomes the ‘5,’ as it lies in the centre of the number line, so does not appear to be visually closer to either side. There is a reason 5 rounds up – what do you think? (Take some thinking time before reading on…).

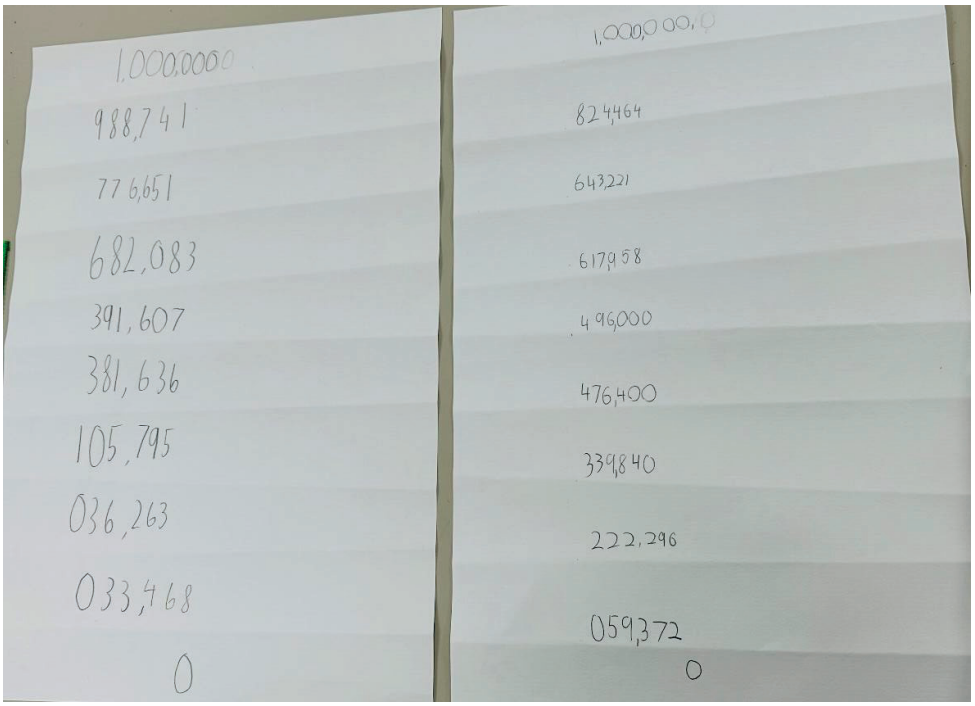
The reason that 5 rounds up is not simply by rule/convention – there is a mathematical basis for it. How many digits are there? There are 10 digits, including zero. So if we count on one hand – 5 digits go down, and 5 go up. That is an even or fair share for situations in life when we round down, and others when we round up.



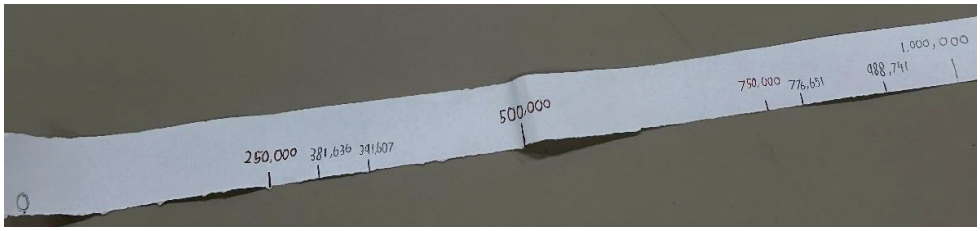
Warm-up Games

Warm-ups	Second phase place value
Climb the Ladder **Focus: Ordering numbers and placing them strategically on vertical number lines**	 Students fold an A4 page into tenths, twelfths (thirds, then half and half again), or eighths (half, half then half again). Play tenths at first to make the strategy easier to grasp. In the top tenth, write in '1 000 000.' In the bottom tenth, write '0.' Students then roll 6 x 10-sided dice (or pull 6 cards with picture cards removed) and place the number formed somewhere on the their board. Students can rearrange the dice, before writing the number on their board, but the numbers must be placed in order and **are locked in once written.**

The goal is to place numbers in order so that the full ‘ladder’ is complete. If they roll a number that cannot be placed in their ladder, they miss a turn.



At the end of each game, mark all the numbers they rolled on a number line from 0 to 100 000 (drawing ‘0’ at the start of the A4 page and ‘1 000 000’ at the end).



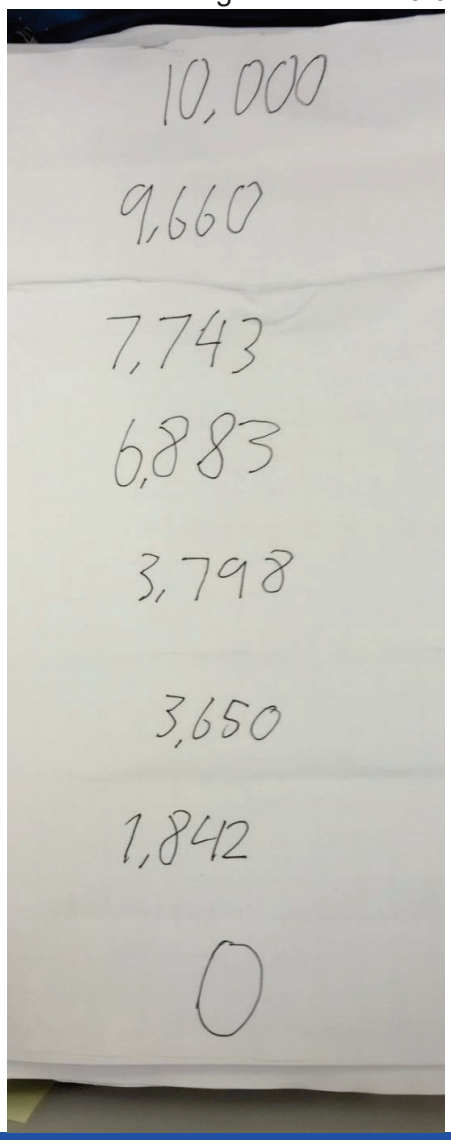
Check where each number should have actually gone by multiplying each number by 0.00021 using a calculator. For example, for 34 065 x 0.00021, it would go at the 7.15cm mark from the left-hand side of the page. In this way, students can reflect on whether they placed two numbers on their vertical number line too close together. That may be the reason that it was more difficult for them (compared to their partner) to finish the ladder without missing too many turns.

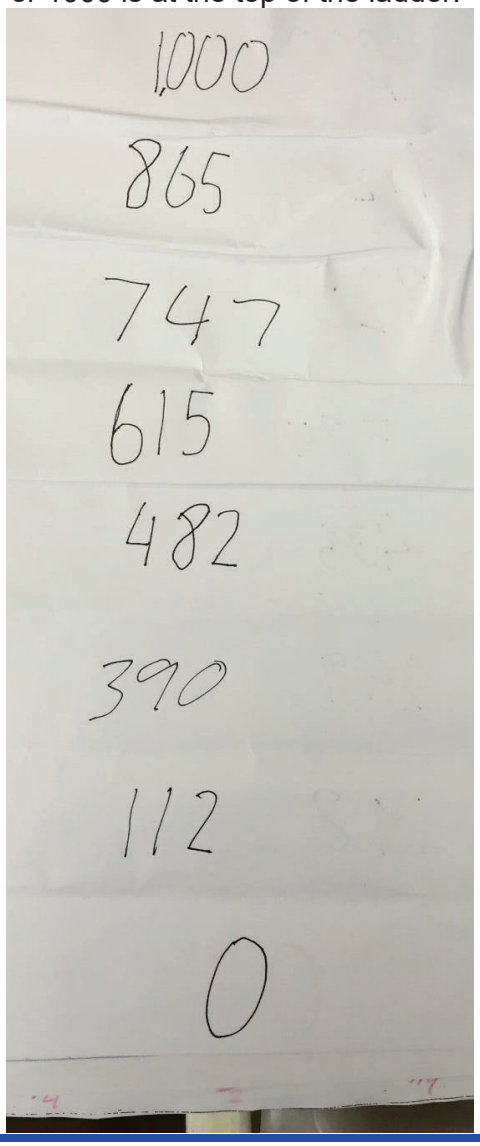
Likewise, if two numbers were placed next to each on the ladder but were very far apart, it may have been that there was too large a gap between them, which also could result in many wasted turns.

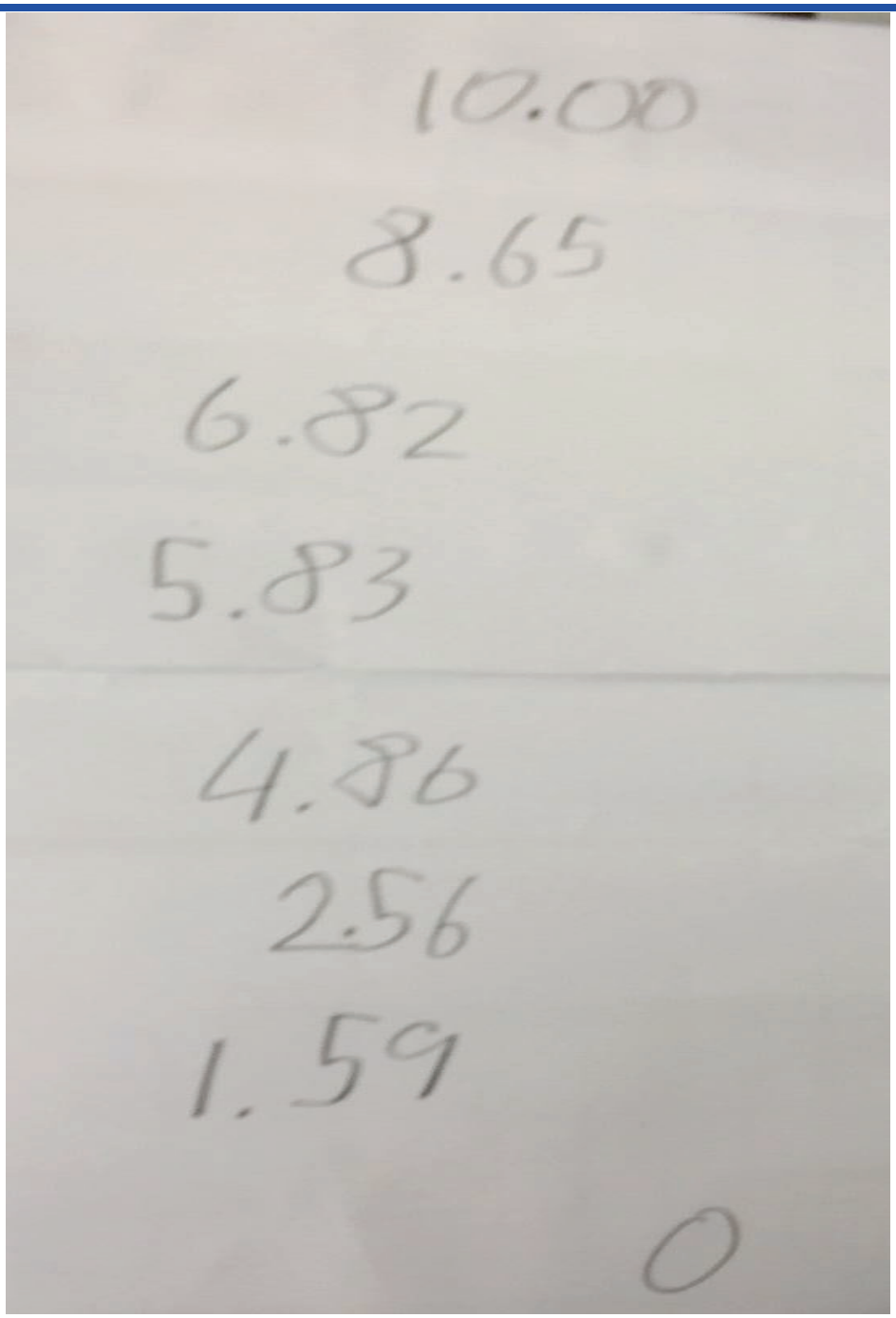
In this sense, this game involves more strategy and less chance to play and successfully complete first.

Student work sample – support versions

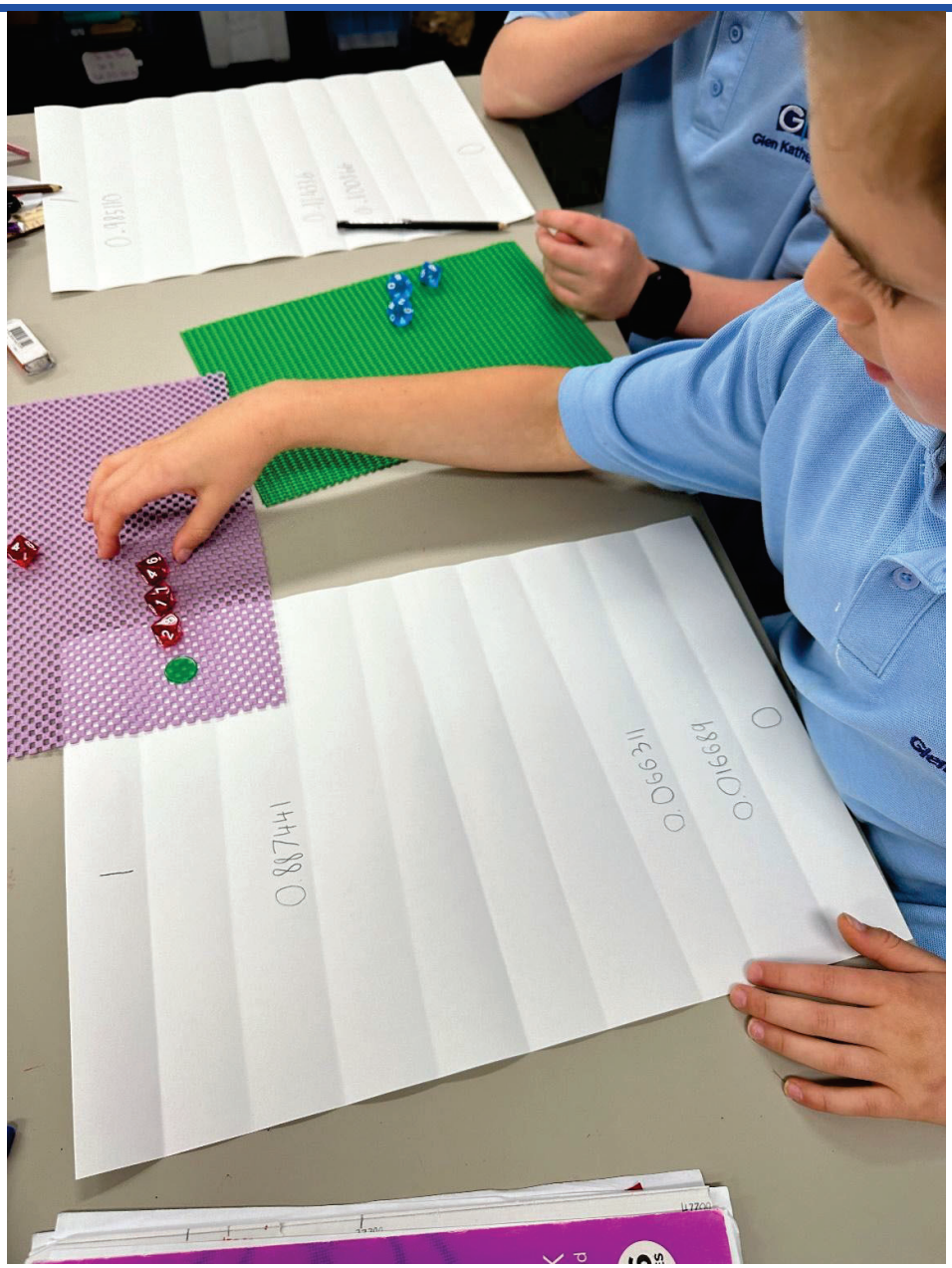
Version of the game where 10 000 or 1000 is at the top of the ladder:



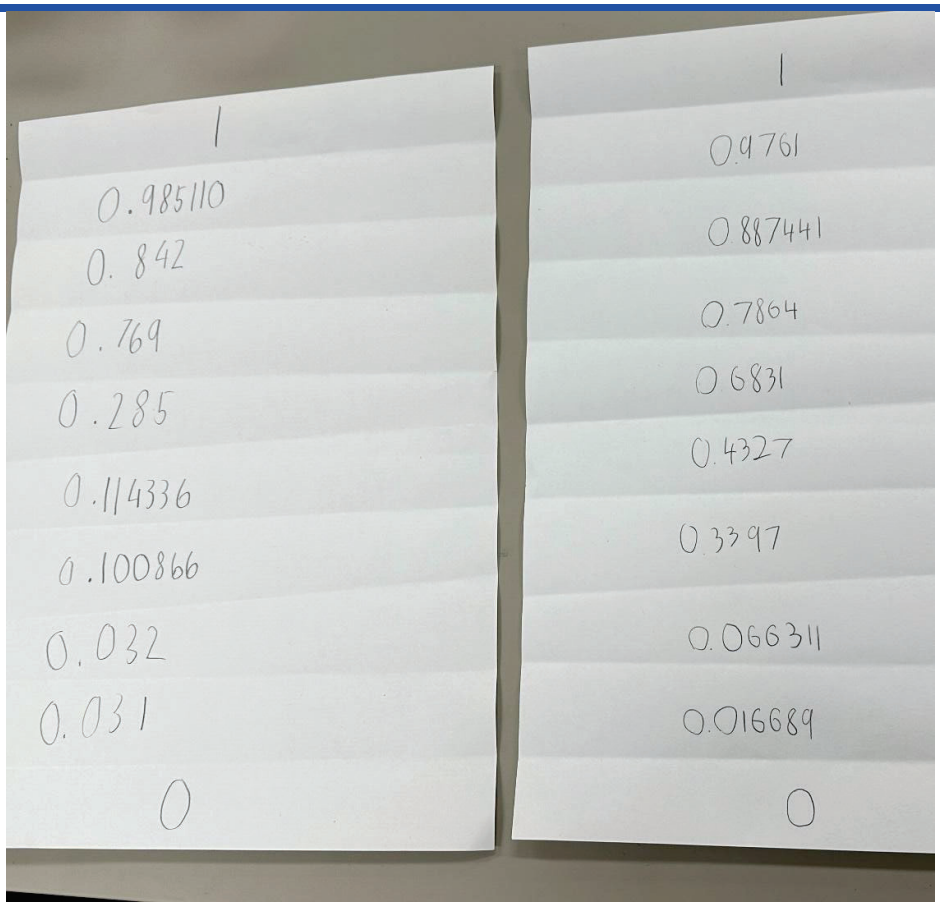




Extension student work sample
Decimal version with 10.00 at the top of the ladder.



Students working on 1 to 0, placing decimals between by rolling dice. Students were able to vary the number of dice they rolled and used a counter as their decimal point.



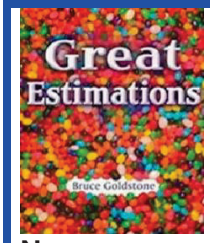
Extension work sample with 1 at the top and 0 at the bottom, with a counter as the decimal point and students able to roll a variable number of dice, depending on their strategy for each turn.

Place Value
Year 5B
Lesson 1

Estimation Containers

Learning intention: Estimate the volume and/or capacity of a container within 25% accuracy.

Maths vocabulary: estimate (thinking guess), capacity (how much liquid it can hold when full), volume (the amount of 3-dimensional space that is taken up, often measured in cubes), millilitres, litres, cubic centimetres, 25% (half of half strategy)



Numeracy picture book link – *Great Estimations:* https://www.youtube.com/watch?v=xEXBPWQvp10

Real-life link: Careers that rely heavily on estimation https://www.youtube.com/watch?v=pxUXwLdRGrs

Estimation 180 website: https://estimation180.com/days/

Lesson summary: Students create estimation containers at their desk, measuring the capacity of their given container. Students then gallery walk to estimate all containers in the room, aiming to be within 25% container's actual capacity.

Materials:

- Collection of containers. Include ones with mL markings to ensure students can use these to measure answer for their given container.
- Access to a tap or drink bottles for students to use water.
- Place value blocks for students to calculate volume, including tens and hundreds, so students can work it out without having to count by ones.
- Post-it notes.

Best set-up: Students create their own container, measuring its capacity or volume (half the students should use water, and the other half use cubes). Then gallery walk to estimate the capacity of their classmates' containers.

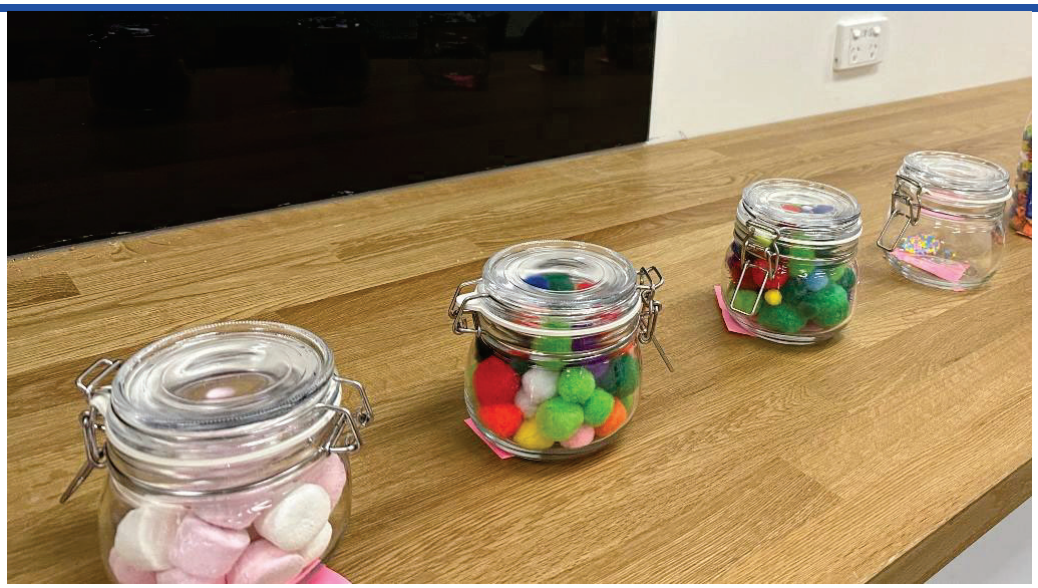
Shopping carts (97): https://estimation180.com/day-135/

Marshmallow jar (44): https://estimation180.com/day-117/

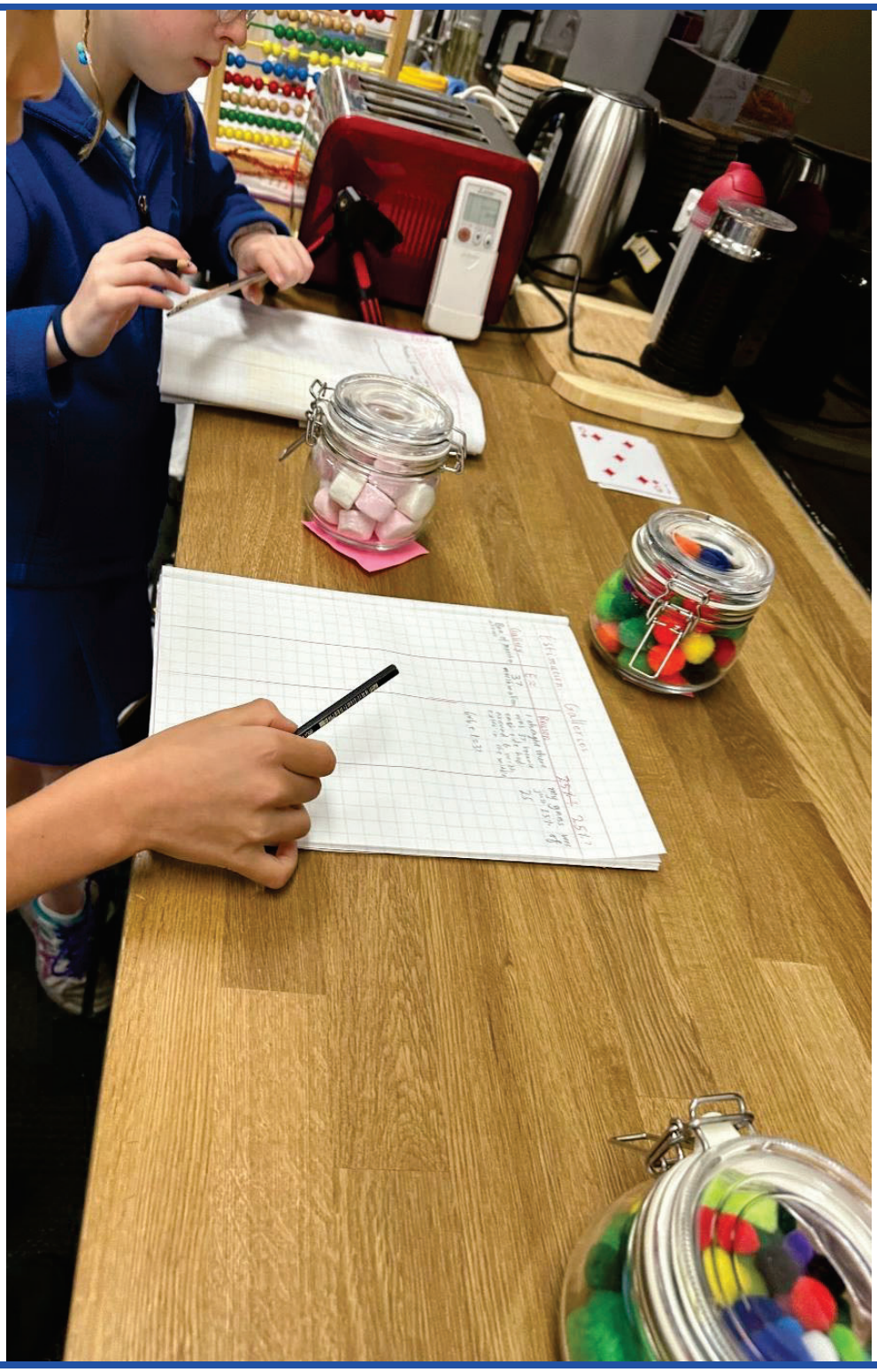
Eggs (93) https://estimation180.com/day-122/

Pages in two different books (first book 183, second book 174): https://estimation180.com/day-105/ and http://www.estimation180.com/day-106/

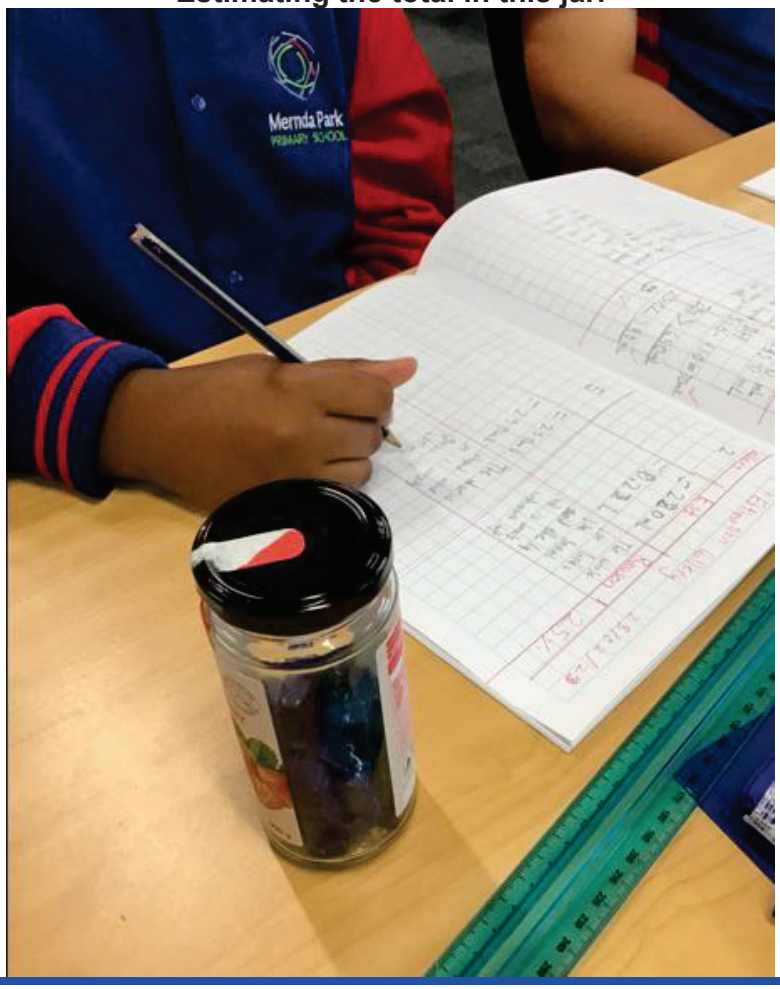
Lego pieces (55): https://estimation180.com/day-193/

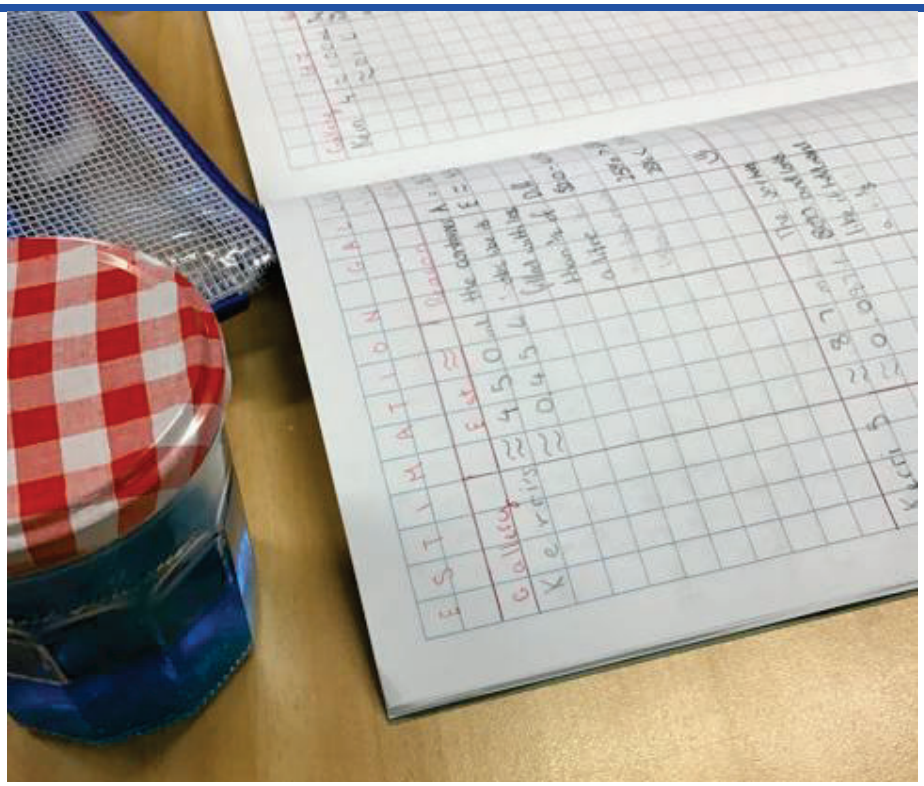


Classroom tip: Galleries using water are particularly quick and easy to set up, but it is important to start with larger objects like giant pompoms or marshmallows first, allowing many students at the start of the session to build confidence and the receive immediate feedback on the reasonableness of their estimates at a whole-class level first.

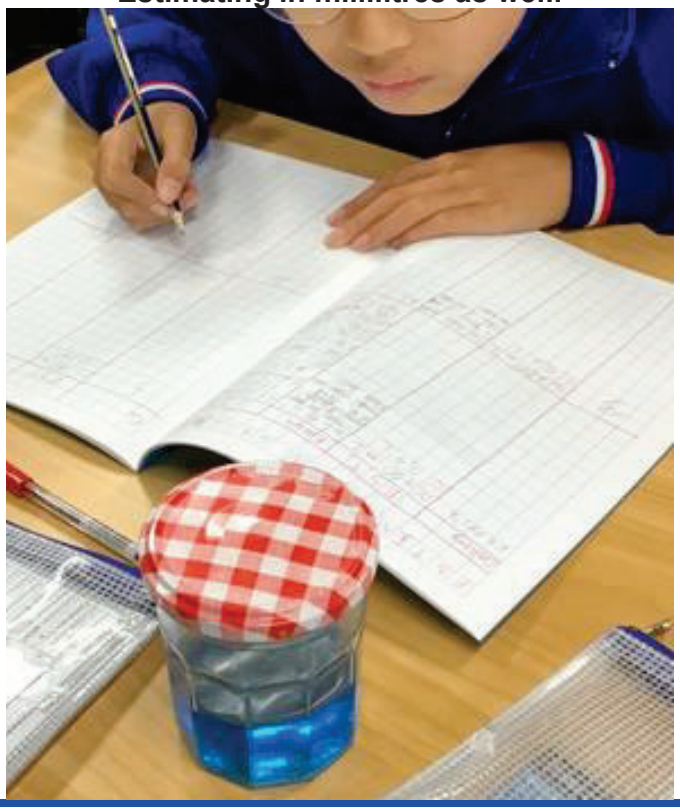


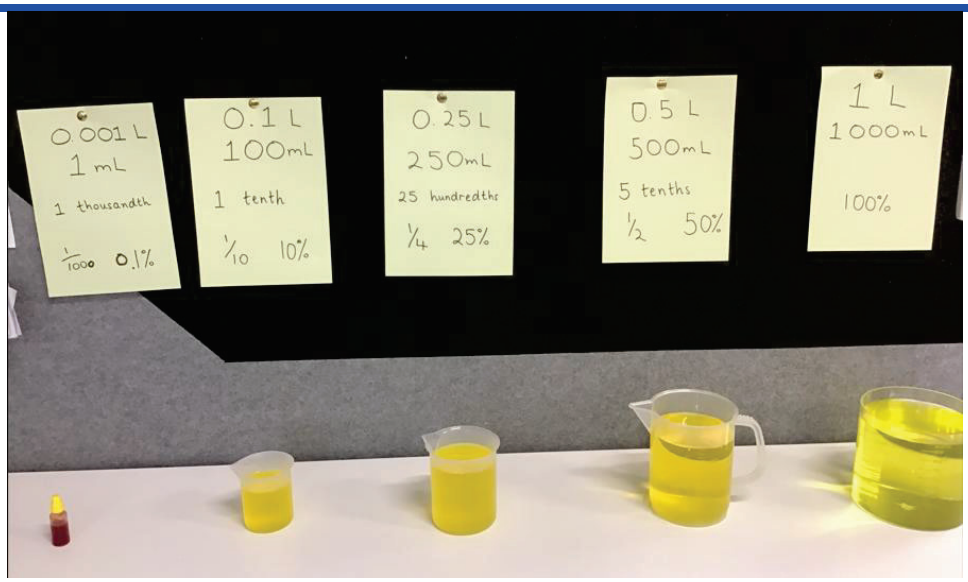
Estimating the total in this jar.





Estimating in millilitres as well.

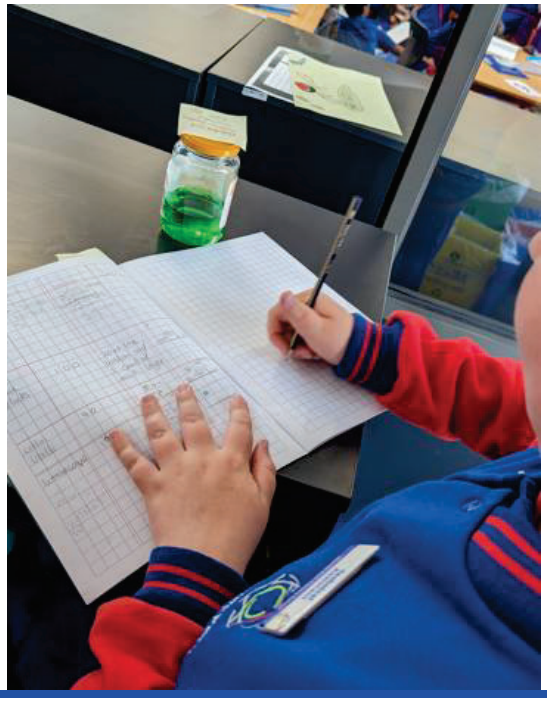


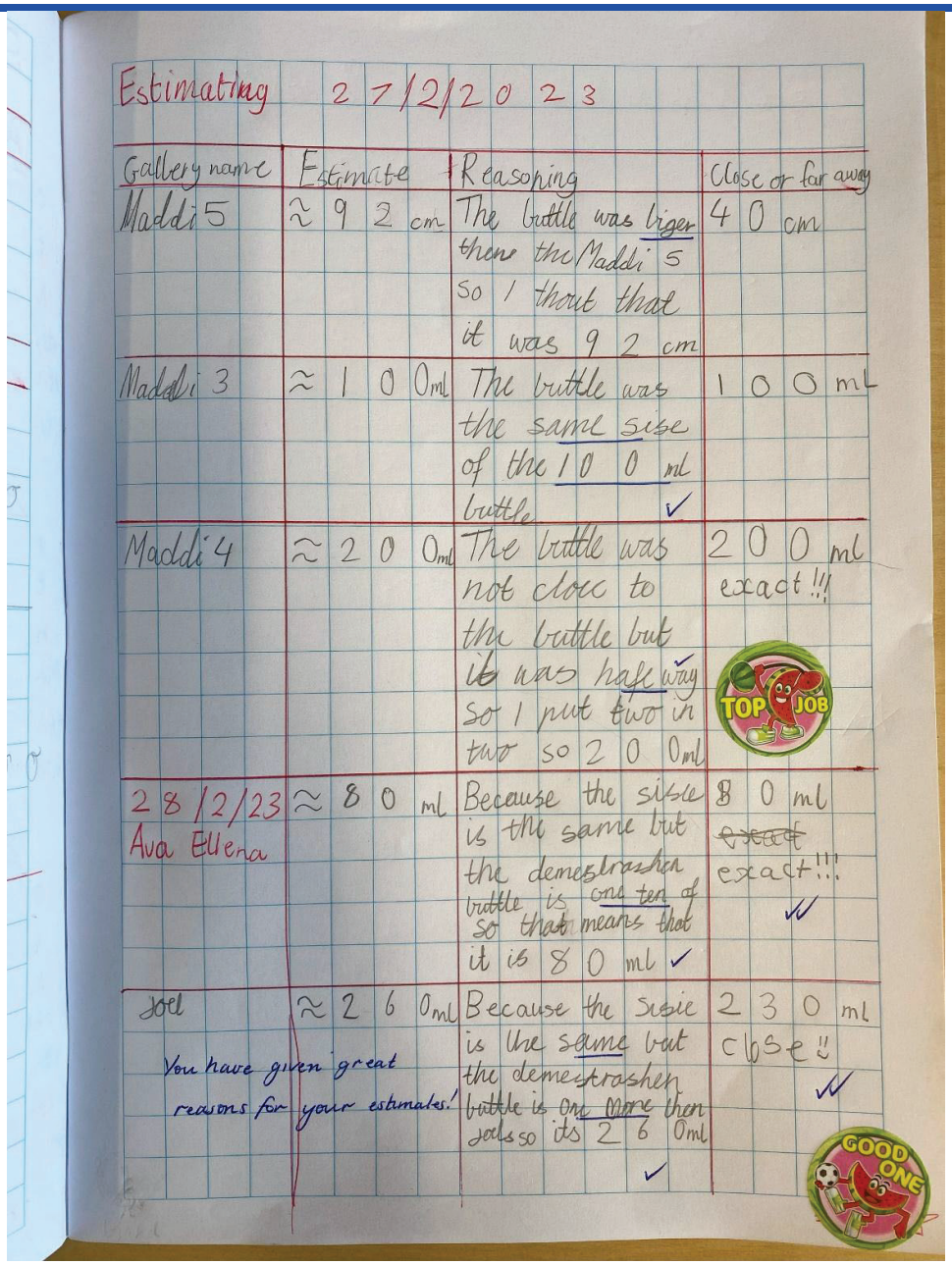


Visual benchmarks at the front of the room to assist students to estimate in millilitres and by fractions of a litre – with great thanks to Mernda Park PS.

If this is one litre…what do you think this container holds?

Excellent strategy by the leaders at Mernda Park PS to use cordial to make water colourful and visually clear.





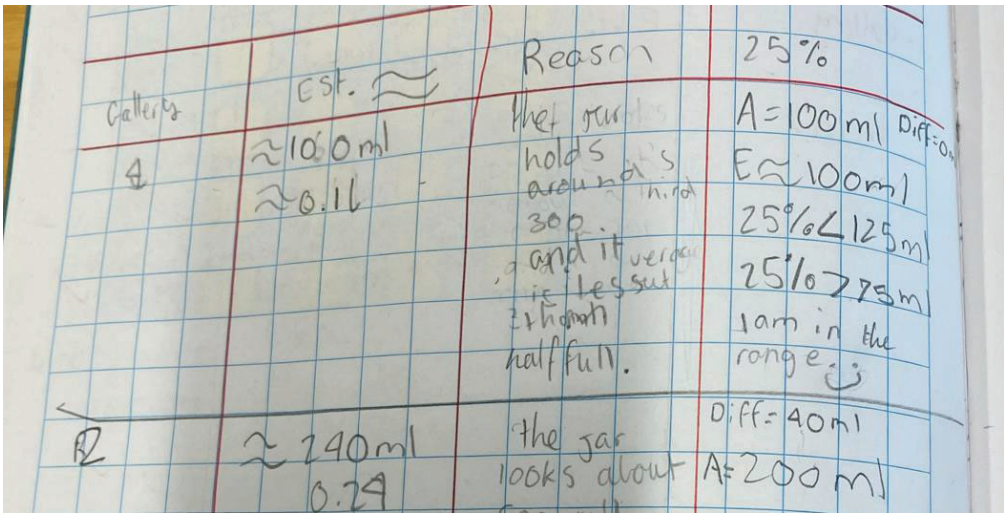
Estimating 27/2/2023

Gallery name	Estimate	Reasoning	Close or far away
Maddi 5	≈ 92 cm	The bottle was liger than the Maddi 5 so I thout that it was 92 cm	40 cm
Maddi 3	≈ 100ml	The bottle was the same sise of the 100 ml bottle ✓	100 ml
Maddi 4	≈ 200ml	The bottle was not close to the bottle but it was half way ✓ so I put two in two so 200ml	200 ml exact!!! TOP JOB
28/2/23 Ava Ellena	≈ 80 ml	Because the sisse is the same but the demestrasher bottle is one ten of so that means that it is 80 ml ✓	80 ml ~~exact~~ exact!!! ✓✓
Joel	≈ 260ml	Because the Susie is the same but the demestrasher bottle is One More then Joels so its 260ml ✓	230 ml close!! ✓✓ GOOD ONE

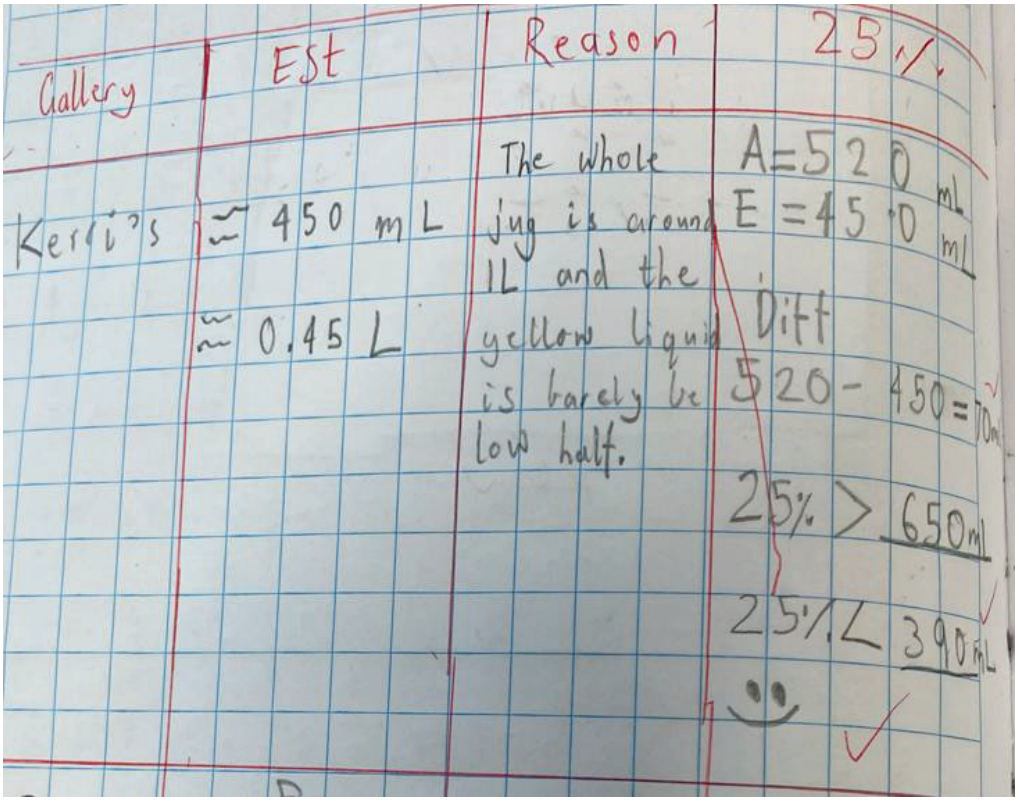
You have given great reasons for your estimates!

Questioning prompts:

- Ask students where their eyes focused – was it on the larger or smaller place values for the blocks and cash? Which places matter the most for estimation purposes?
- Does an estimate need to be 'spot on' accurate? Should it be?
- Which professions use estimation a lot? (Painters for the amount of paint required, builders to quote a job, tree loppers for green waste costs).



Student work samples



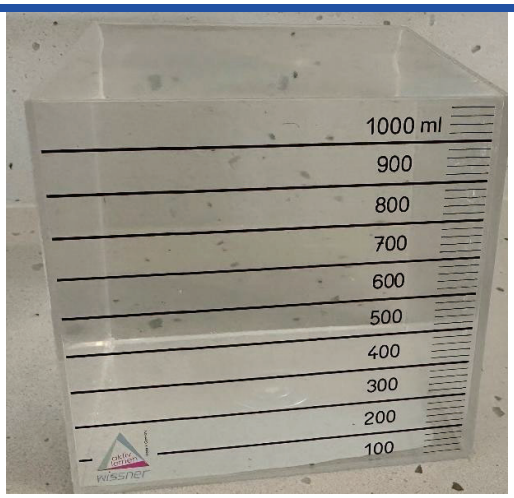
Connecting the ones place value block ($1cm^3$) to millilitres, the hundreds to 100mL and the one thousands cube to 1L (as well as 1kg)

Modelling: Model the meaning of 1mL using a single drop into a container. What would 100mL look like? Show students a 100mL container, or a container filled with 100mL. What would 250mL look like? What about 500mL? What about 1000mL or 1L?

Also model 1 cubic centimetre by connecting this to the ones place value cubes. The ones is a cube, where all three dimensions (length, height and depth) are 1cm, so it is one centimetre cubed.

Finally, connect mL to cm^3 by showing students that if a container can hold 100mL, it can also hold 100cm3 (100 ones cubes). Therefore, to estimate in mL, you can also just think about how many place value blocks (or how many layers of ones) the container could hold if the ones cubes were made of liquid.

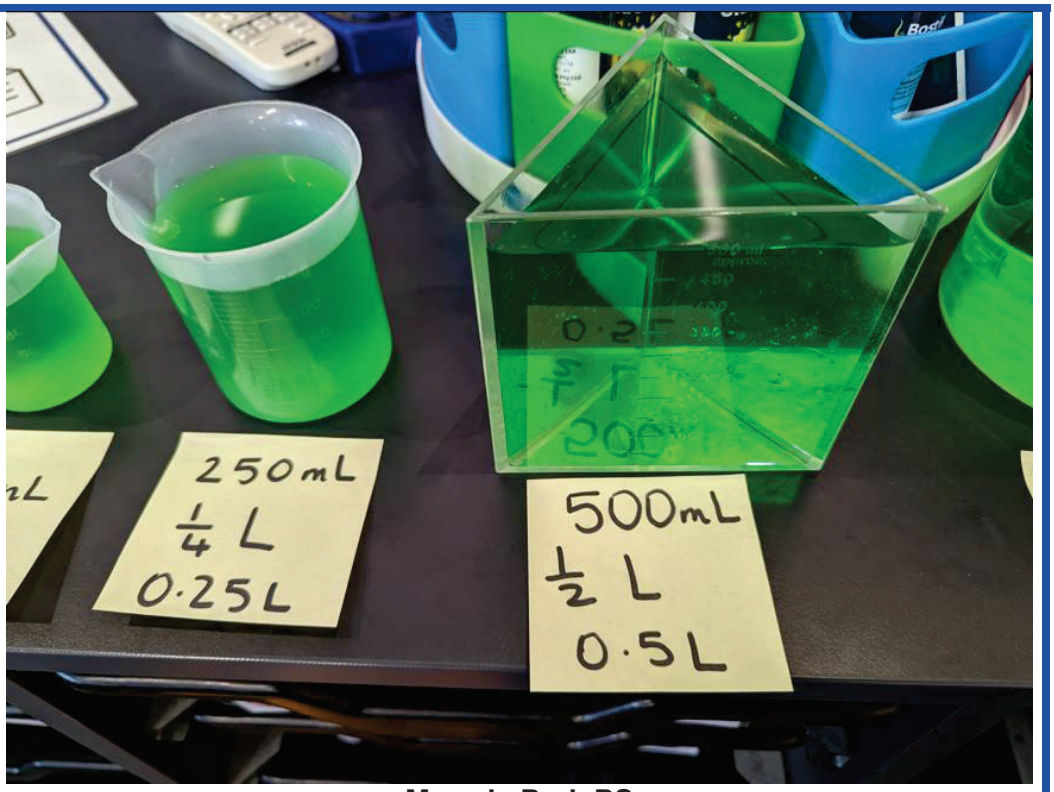
The container is wet as the student just checked that it fit 1000 millilitres or 1 litre of water, and now is checking that it fits 1000 cubic centimetres or 1 thousands cube.



The same container fits 1000mL, and also 1000cm^3.

Beware: Containers that create gaps (like orange juice bottles) will not fit the same number of cubic centimetres as millilitres.

For example, this 500mL juice bottle only fit 300 cubes and the 1L only fit 650 cubes:



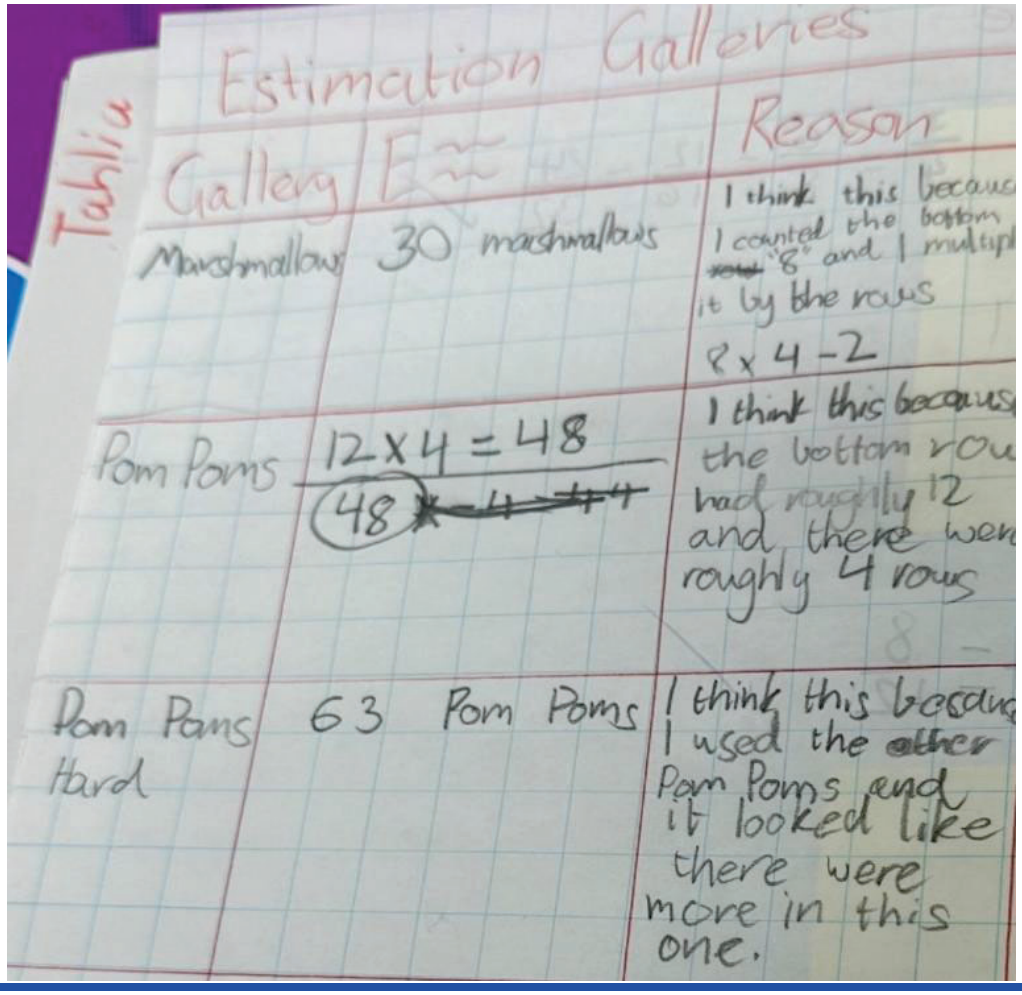
Mernda Park PS

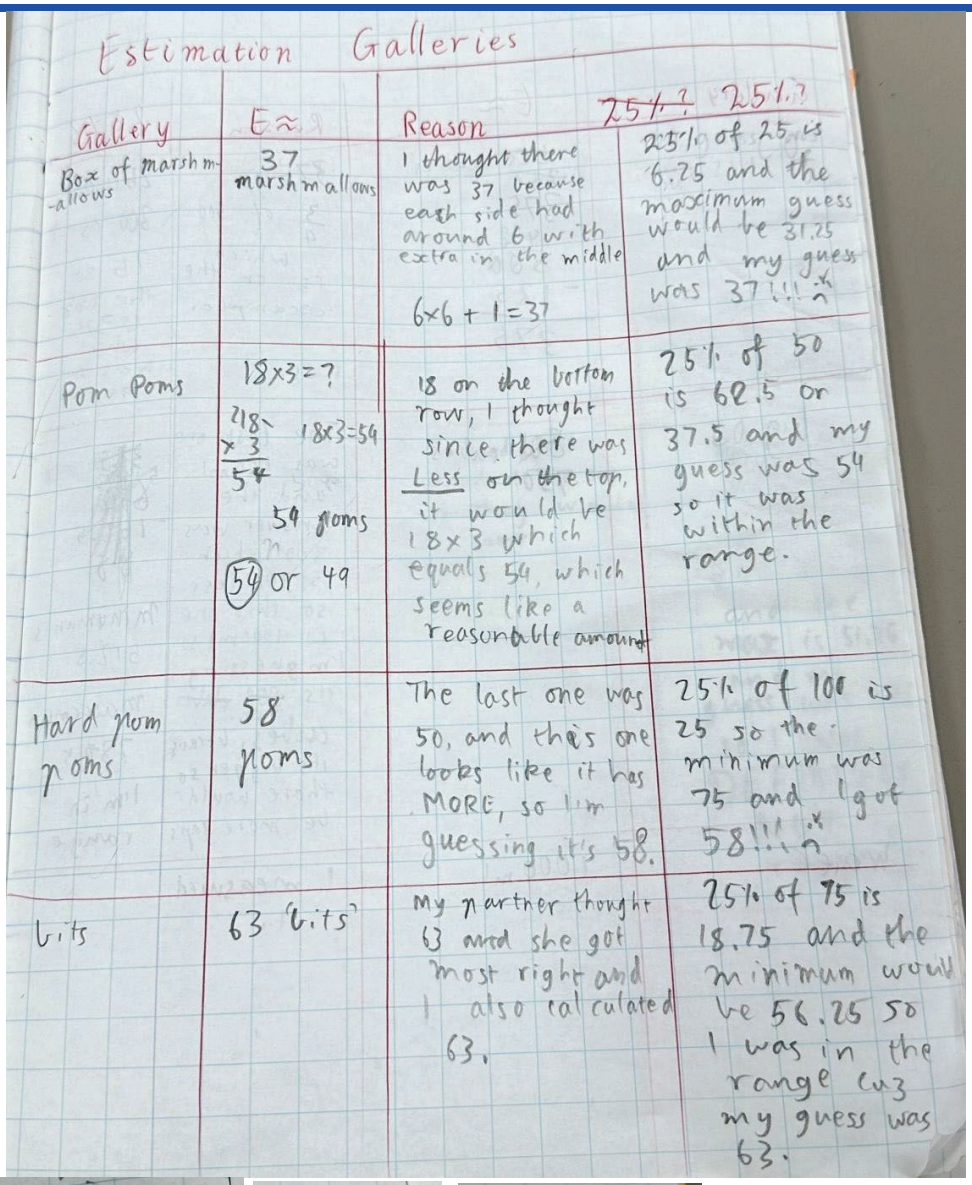
Whole-Class Modelling – Practising Estimation Galleries and the 25% Margin for Error Together

Whole-class practice: Fill up some containers with water and practise estimating as a class. Students must record the reasoning behind their estimates, using 4 columns in their grid books.

Gallery name	Estimate	Reasoning behind your estimate	Answer and within 25%?
Mila's gallery	1200mL or 1.2L	The container is really wide, so could hold almost a one hundred block on its base. I think it is about 12 cubes high (so, 12 x 1 hundred). It looks a bit larger than a thousand cube.	1500mL or 1.5L 😊 (estimate was reasonable) Half of 1500mL is 750mL, half of 750mL is 375mL, so my estimate was within 25% (within 375 of the answer). To work this out, use a half of half strategy.

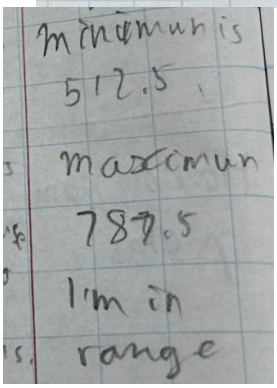
Misconception alert: During the whole-class practice, be sure to highlight the classic misconception of wide and short containers v. skinny and tall containers. Students are often biased towards height and underplay width when estimating and comparing volume and capacity.

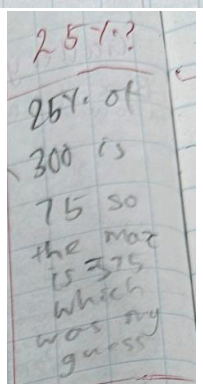


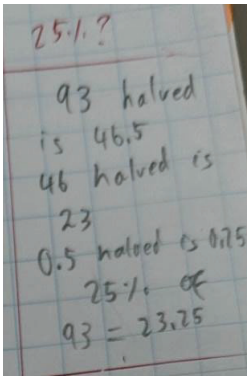


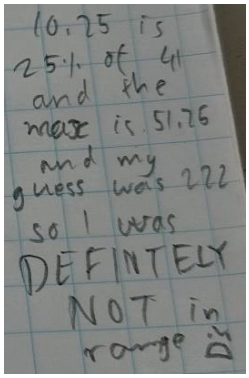
Estimation Galleries

Gallery	E≈	Reason	25%? 25%?
Box of marshmallows	37 marshmallows	I thought there was 37 because each side had around 6 with extra in the middle 6×6 + 1 = 37	25% of 25 is 6.25 and the maximum guess would be 31.25 and my guess was 37!!! ✓
Pom Poms	18×3=? 218 × 3 = 54 54 poms (54) or 49	18 on the bottom row, I thought since there was Less on the top, it would be 18×3 which equals 54, which seems like a reasonable amount	25% of 50 is 62.5 or 37.5 and my guess was 54 so it was within the range.
Hard pom poms	58 poms	The last one was 50, and this one looks like it has MORE, so I'm guessing it's 58.	25% of 100 is 25 so the minimum was 75 and I got 58!!! ✓
bits	63 bits	My partner thought 63 and she got most right and I also calculated 63.	25% of 75 is 18.75 and the minimum would be 56.25 so I was in the range cuz my guess was 63.


minimum is 512.5 maximum 787.5 I'm in range


25%? 25% of 300 is 75 so the max is 375 which was my guess


25%? 93 halved is 46.5 46 halved is 23 0.5 halved is 0.25 25% of 93 = 23.25


10.25 is 25% of 41 and the max is 51.25 and my guess was 222 so I was DEFINTELY NOT in range

Example of Year 4 extension student recording, including the within 25% element

Student Estimation Galleries

Once most students estimates are frequently within 25% of the answer, set them the task of creating their own container for their classmates to estimate. Half the class should use water and half cubes (girls use water, boys use cubes; then switch for the next session).

For cubes, use tens, hundreds and even thousands blocks where possible (not just ones). Fill each container to the very top (its capacity). Name their gallery with a post-it note (e.g. Mila's gallery).

Once students have created their container, filled with water or cubes, work out the answer to its capacity and record this on a post-it note stuck under the desk or hidden under the grip mat. This provides immediate feedback for other students, after they estimate and record their reasoning, which will then improve their estimates going forward to new galleries.

Reflection: What weighs more, 1L or 1000cm^3? Place a 1L container on one side of a balance scale and a 1 thousand block on the other. ***Note:*** Test this before students enter the classroom to ensure the 1 thousand block is correctly weighted (use a wooden one that is not aged). They are equal – both weight 1kg! What weighs more, 1mL of water or 1 cm^3 (1 cm cube, a ones cube)? They weigh the exact same – 1 gram! What will be equal to a tens block? What about the hundreds block?

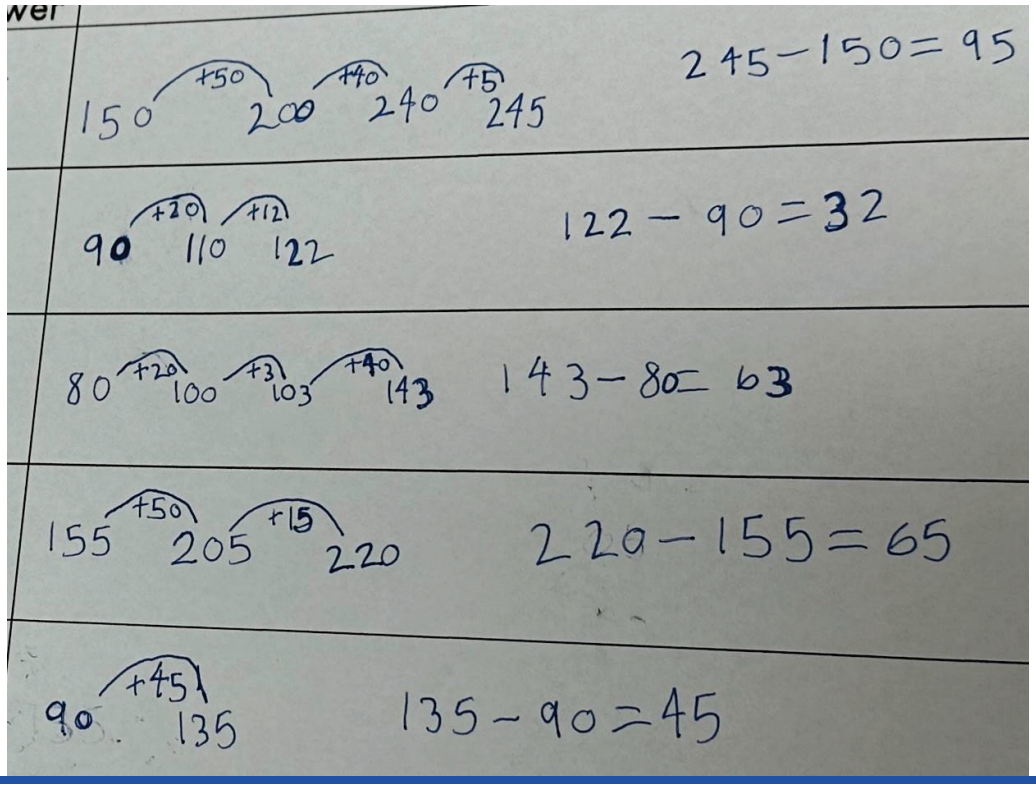
Support 1 – Ignore the percentages: Remove the percentage element, in terms of working out how close their estimate was. Instead, allow them to decide whether the estimate was 'close' or 'way off' – students are usually quite harsh on themselves, so ensure they are allowing themselves a margin of error.

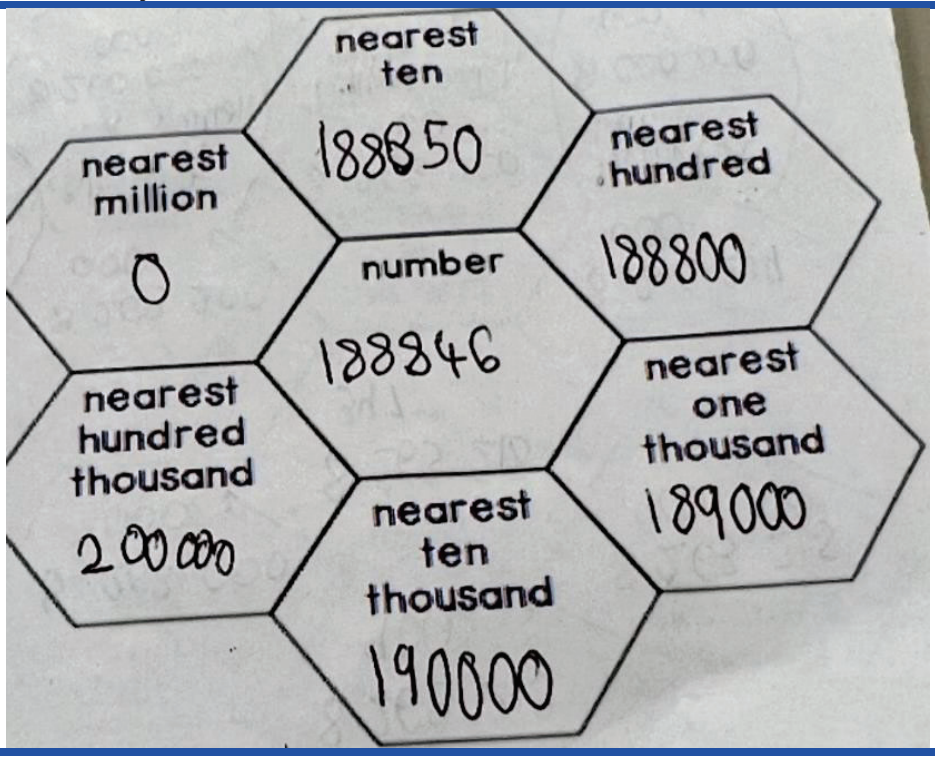
Support 2 – Use benchmark containers as reference points: Carry around a few example containers with them – one container that can carry exactly 100mL and 100cm^3, another that can hold 250, and another that holds 1000. These could be placed at the front for other students to use as benchmarks, but support students should have their own that they can carry to each gallery to use as supportive references.

Extension 1: Place a goal of being within 12.5% or 15%, thereby making the percentage calculation more challenging as well.

Extension 2: Use a mental strategy (such as a jump strategy) to work out how far off their estimate was from the actual capacity.

Student work sample for jump the difference strategy recording



Place Value Year 5B Lesson 3	**Rounding Parkour** **Learning intention: Round numbers to any place value by thinking *"What is it closer to?"*** **Maths vocabulary: round (what is it closer to), place value, number line, halfway (1/2), quarter (1/4), three quarter (3/4), benchmarks**
YouTube hook – Parkour: https://www.youtube.com/watch?v=NX7QNWEGcNI&t=3s and https://www.youtube.com/watch?v=AepwD2PpDO0	**Lesson summary: Students jump to each part of the rounding parkour templates with a mini figurine, rounding a number to different place values to earn 'parkour' points. Students also use a piece of paper beside the template to create mini number lines to assist the rounding for every place value, one-at-a-time. After completing the template, students place their number on a 1m number line that spans from zero to one million, or 0 to 1 hundred thousand (or less) for support pairs.** **Materials:** • Rounding parkour templates and plain A4 paper. • Students can bring from home – a mini figurine to act as their parkour 'player' or 'character' during these sessions. *Alternative:* Pokémon counters, Lego people or similar. • Playing cards (remove all picture cards, keep aces as ones). • 1m string/rulers/pipes. **Best set-up:** Whole-class model at a desk with materials, including modelling the number line folding and the template with the class doing a few examples together during the first session to practice both simultaneously.
	

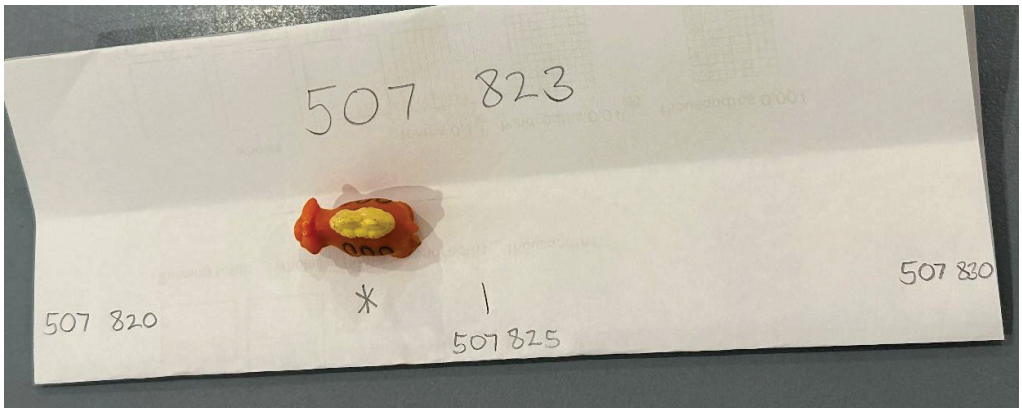
Number Line Strategy – "What is it closer to?"

Use a new piece of white A4 paper for each number that students are trying to round. Fold the page into eighths – half, half, half, all horizontally. These will become the number lines for each place value you are trying to round to.

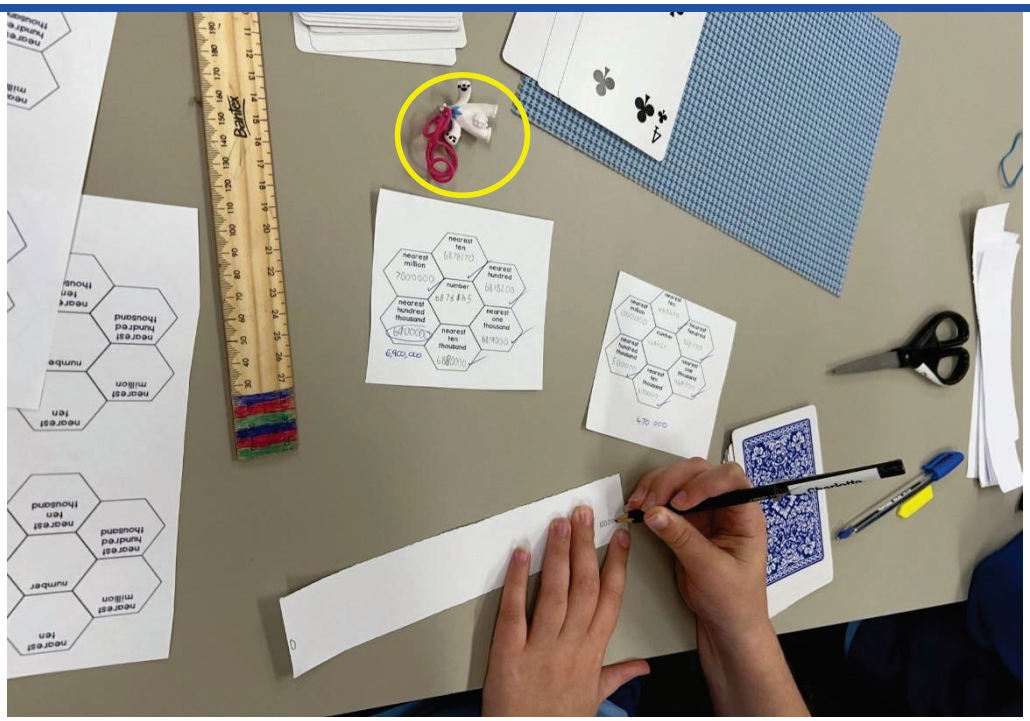
Pull 6 playing cards in any order (use less cards if students are not ready for all place values as yet) to create a starting number. In the top row (first fold), write the number as the heading – 507 823.

For the second row, try rounding to the nearest ten. For example, when rounding 507 823 to the nearest ten, write the ten the number is currently in to the left and the next ten to the right.

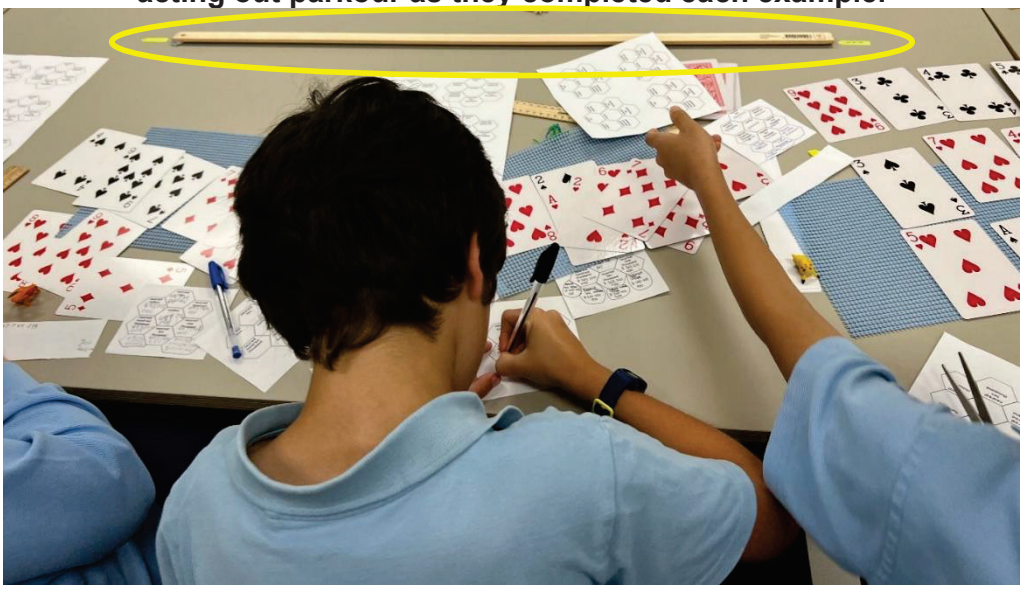
Then mark the halfway point (half of that ten). Mark the quarter and three quarter benchmarks also if they help, particularly for the larger place values. Then mark the number along the line and draw an arrow towards which ten it is closer to. Use your parkour player (mini figurine) to check – if their legs were feeling tired from a huge day of parkour, would they want to jump to *that ten* or *that ten*?



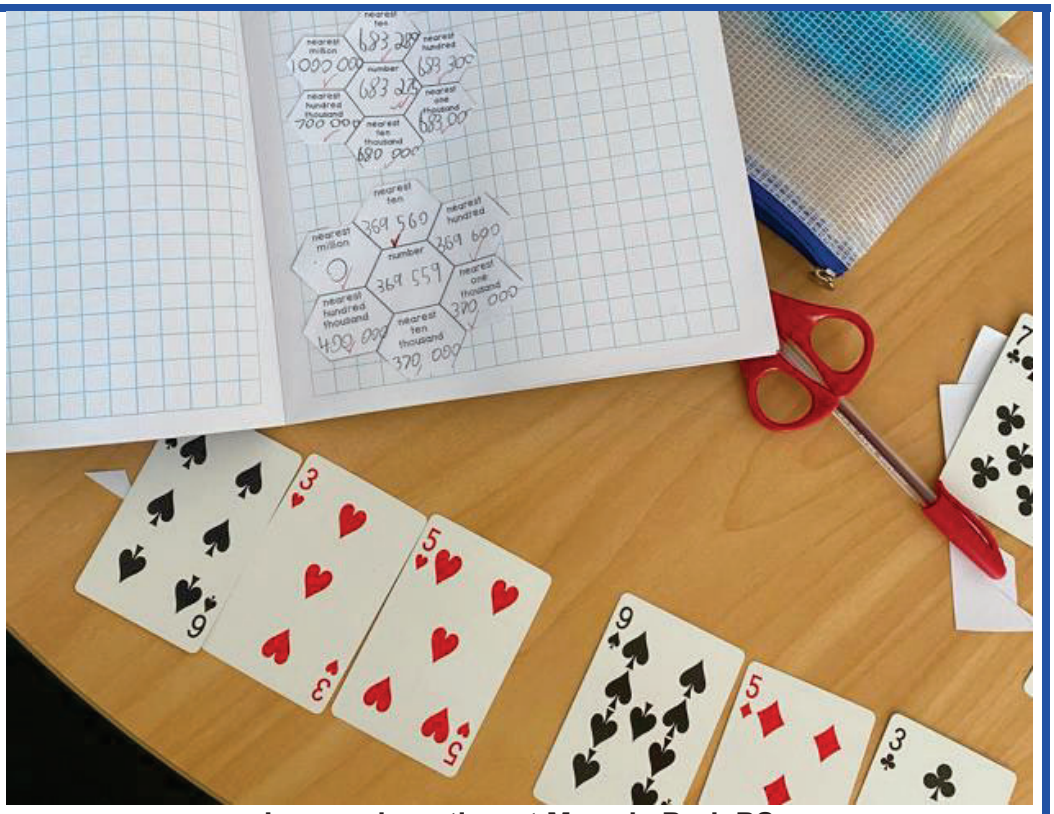
Record this on the parkour template, then move your character to the next hexagon and assist them to solve it using the number lines again. Repeat this process for each place value, emphasising the language 'what is it closer to' and 'what one thousands is the number in' and 'what is the next one thousand?'



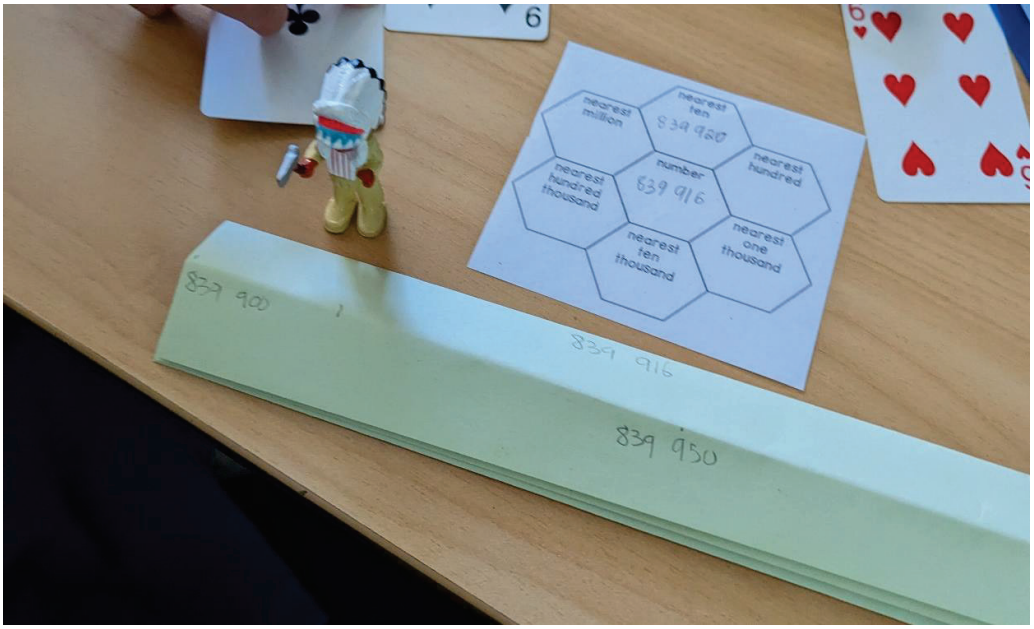
Students using a strip of paper as a number line to determine which number their playing card number is closest to. Students also jumped their mini character (circled above) around the parkour templates, acting out parkour as they completed each example.



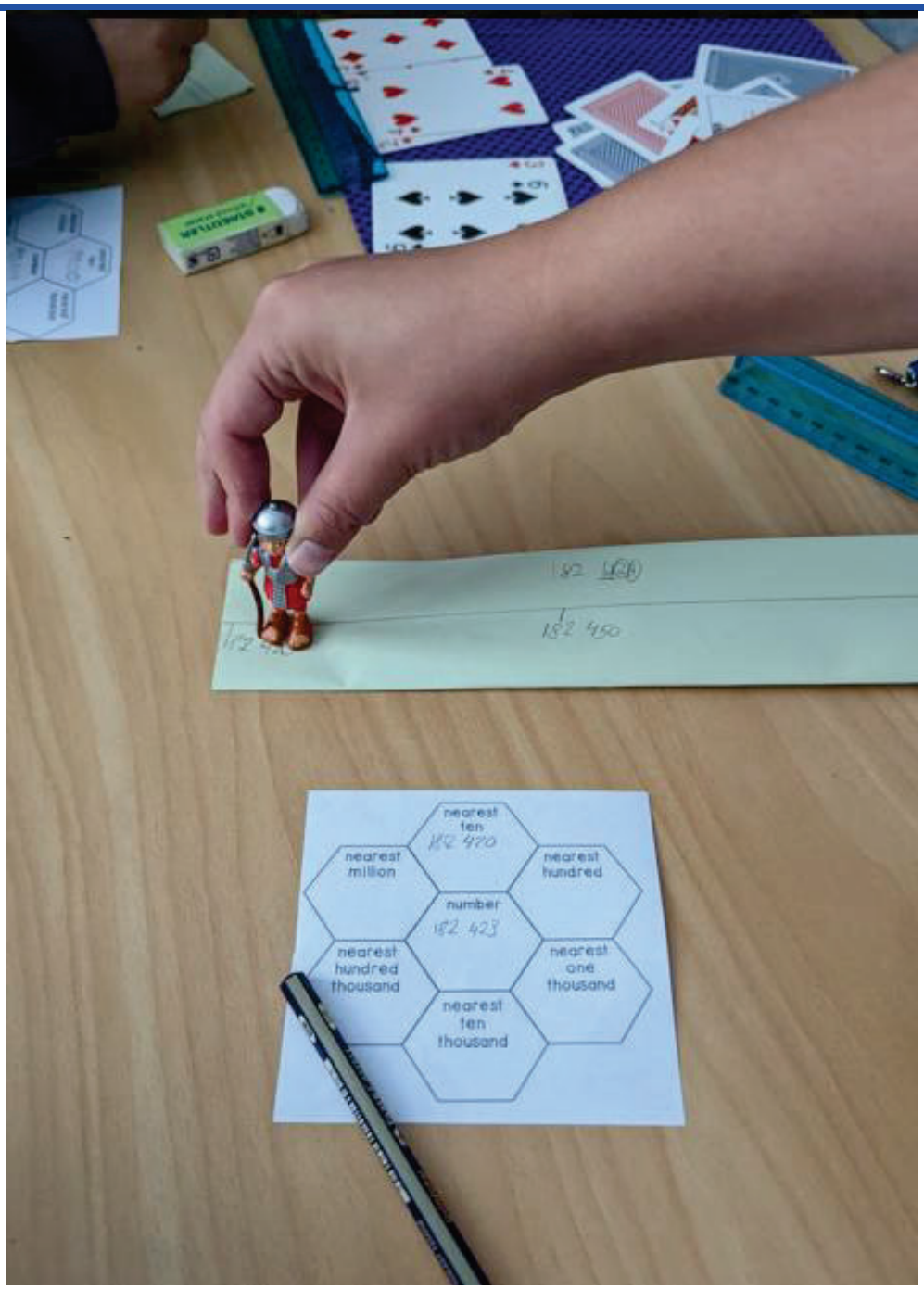
Students placed each number along a 1m number line (1m ruler, 1m string or 1m PVC pipe stuck to the desk). The one metre number lines were set up by challenge level – 0 to 1 billion for extension, 0 to 1 million for mid-range and 0 to 100 000 or 10 000 for support.



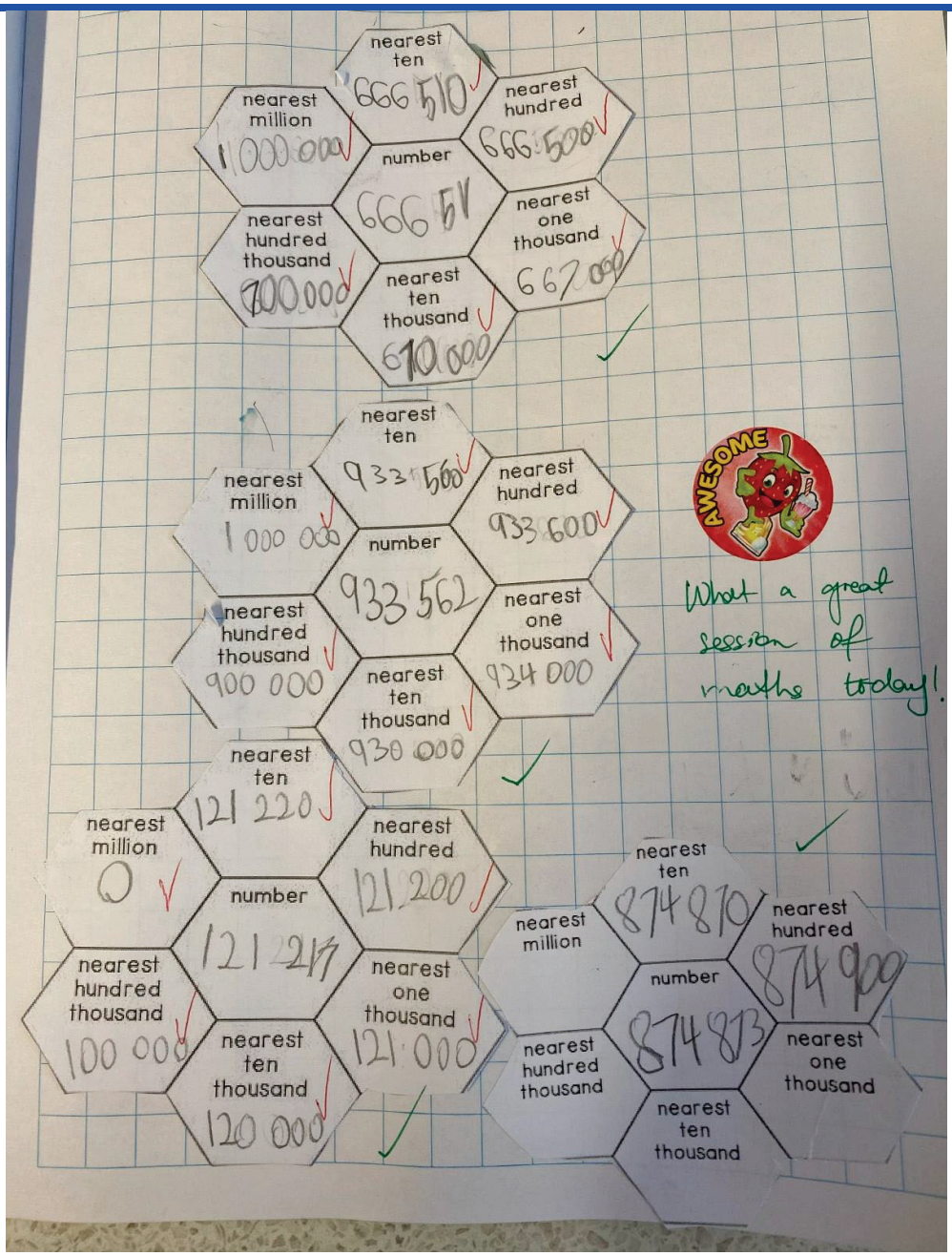
Lesson in-action at Mernda Park PS



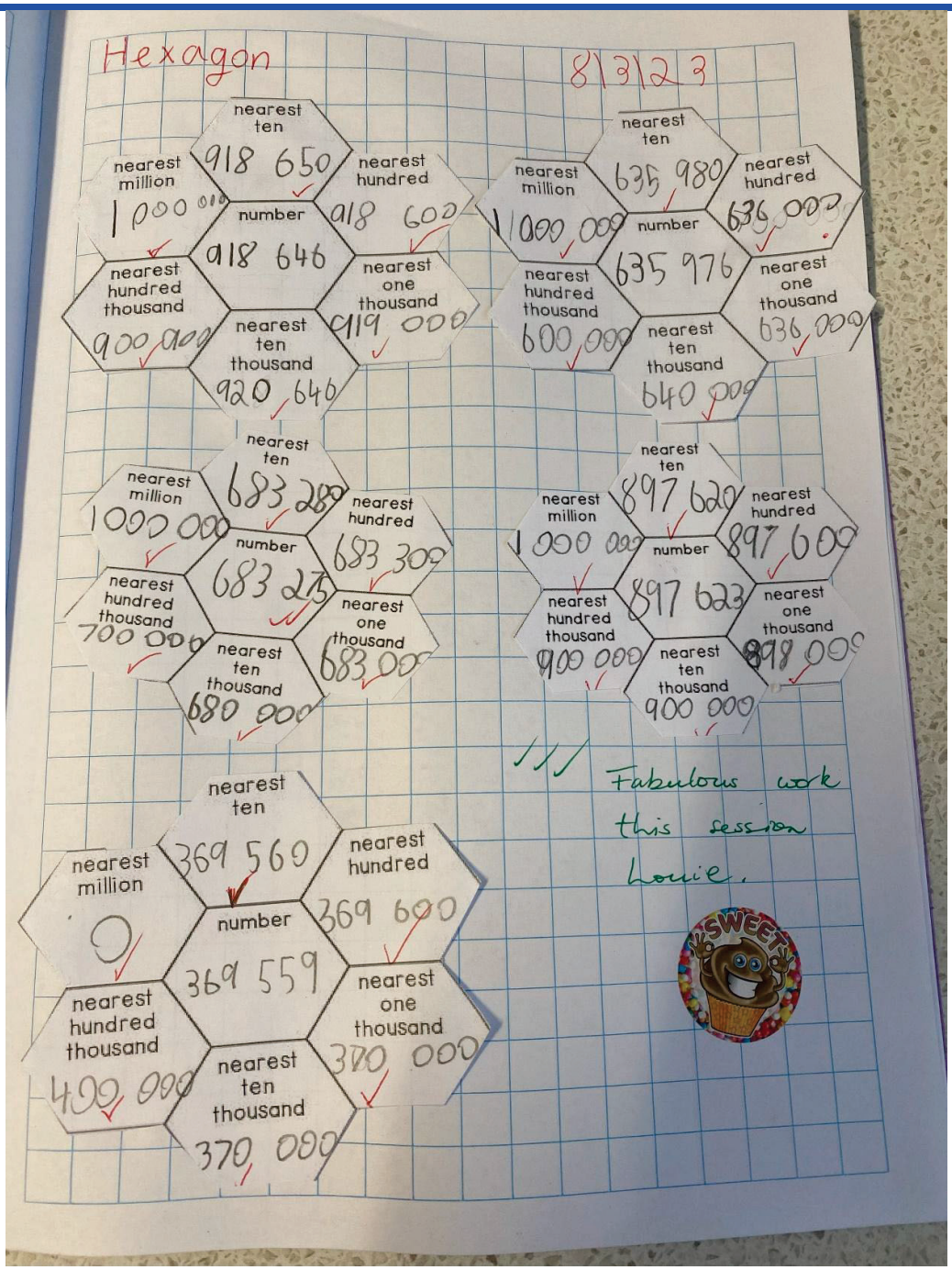
Bringing the parkour hook alive with mini figurines in use along the number lines, to literally jump closer to the safer platform.



Rounding parkour – jump to the closer round number!

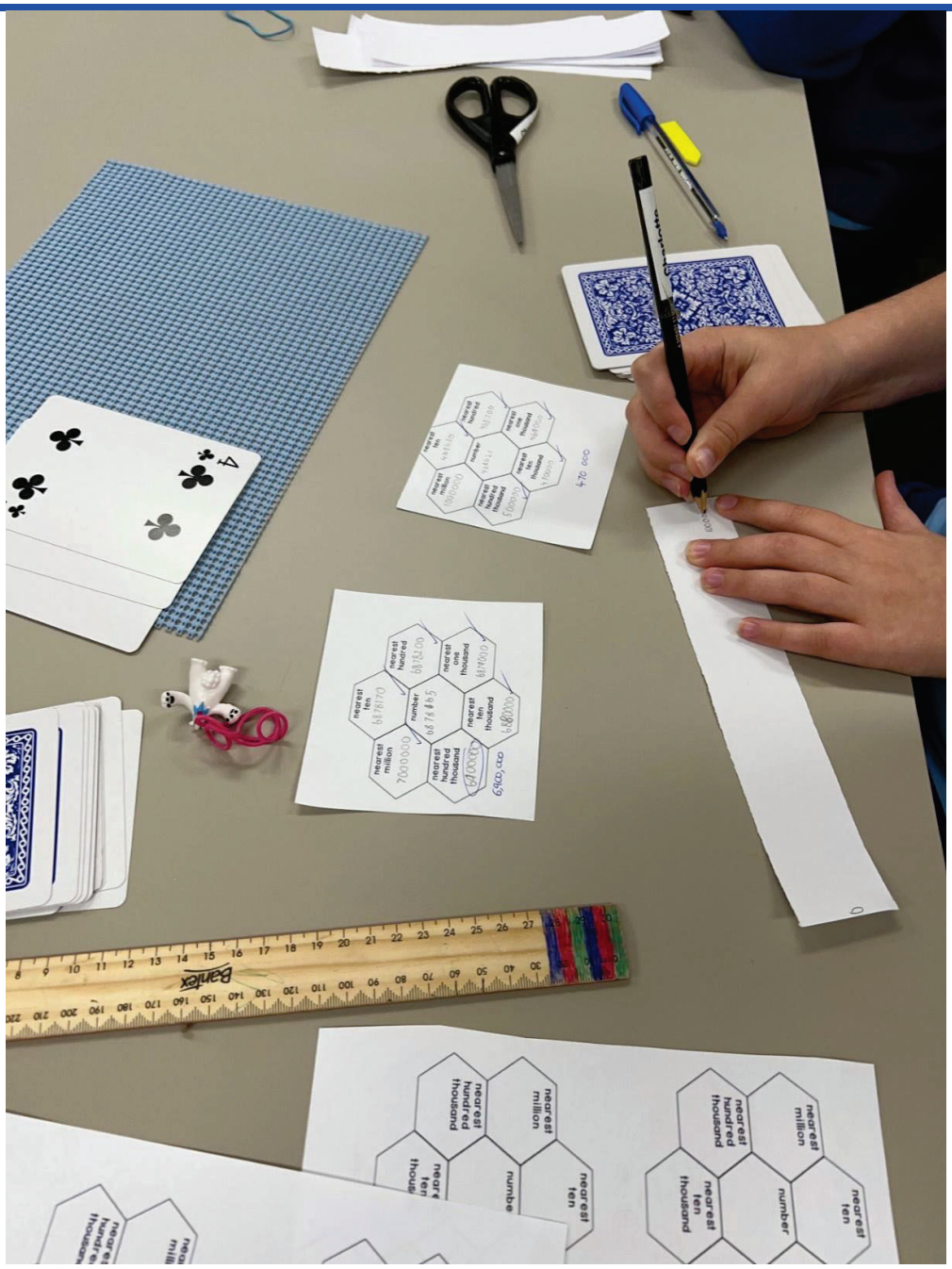


Mernda Park PS student work sample

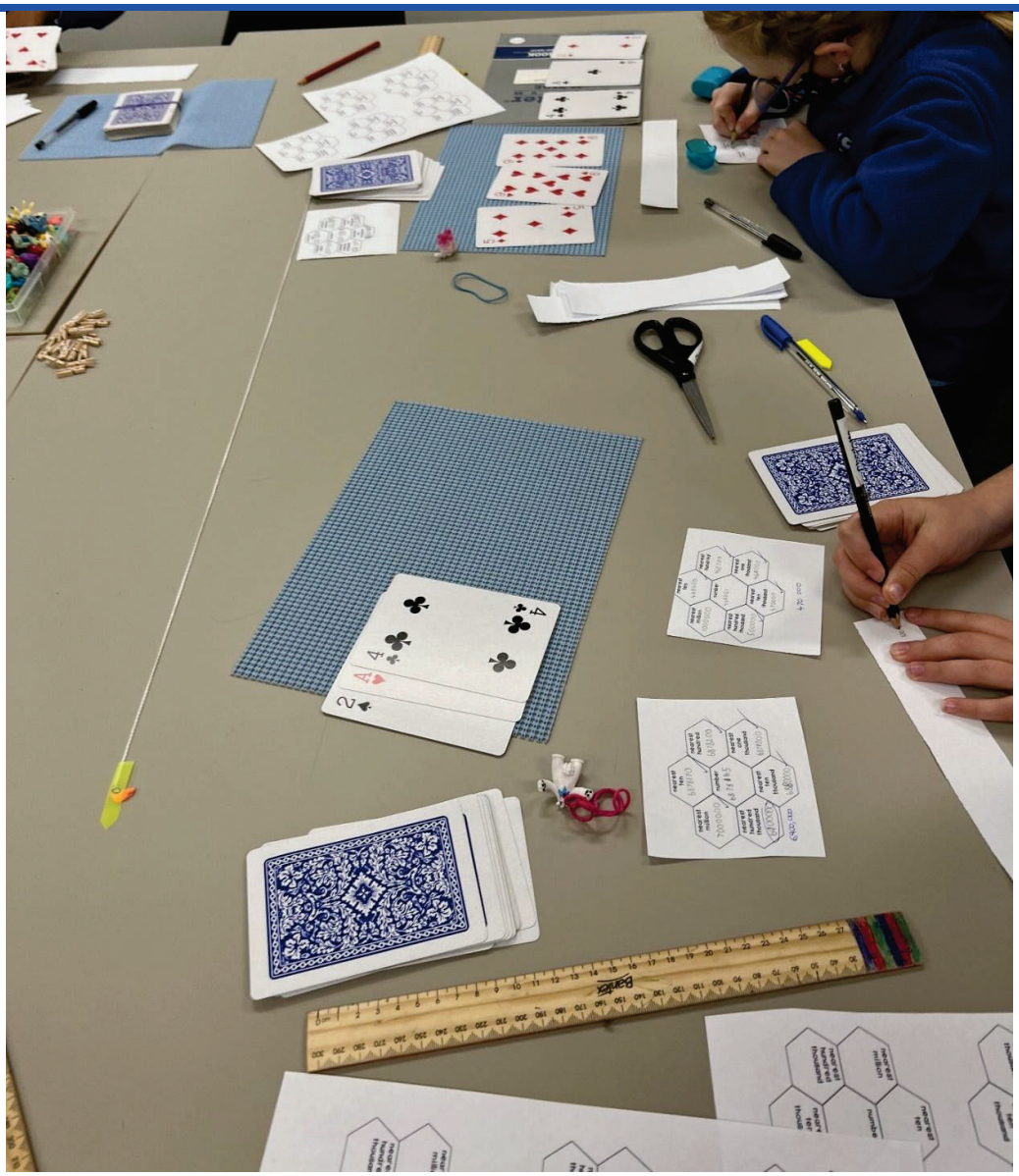


Mernda Park PS student work sample

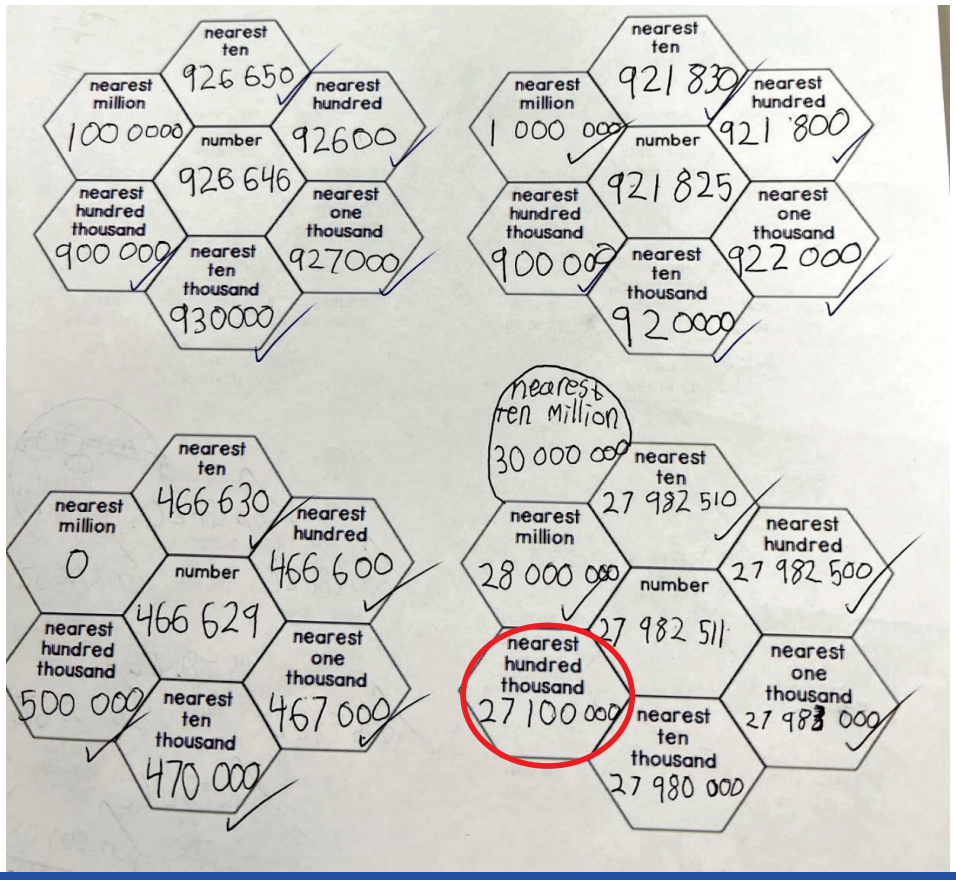
Lesson in action

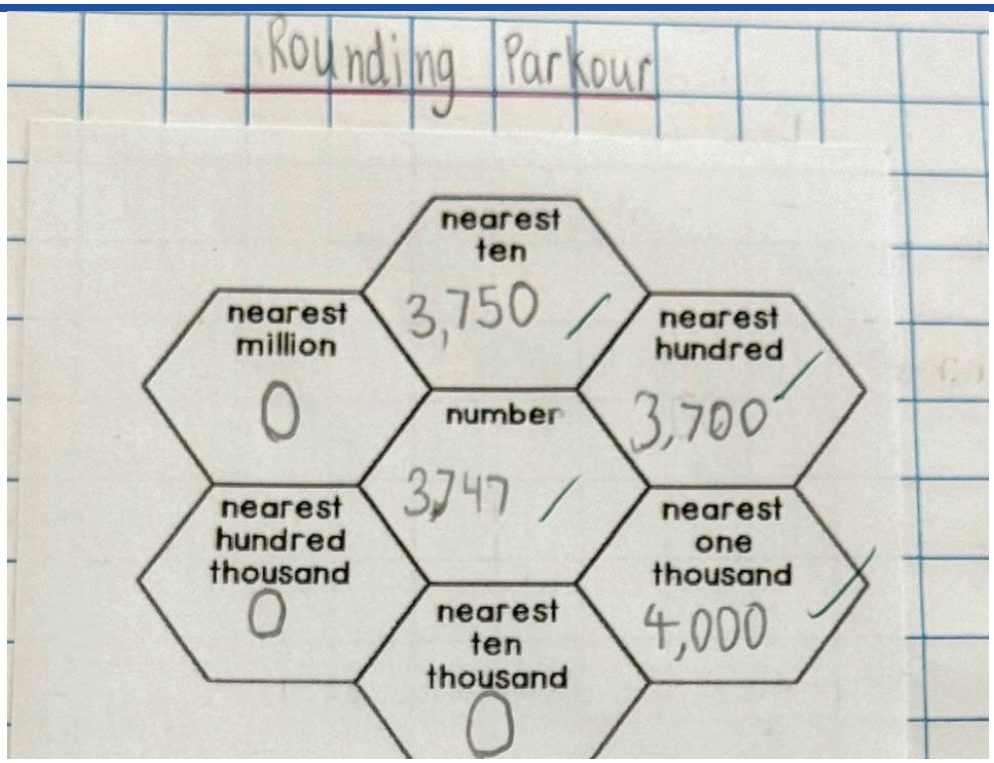


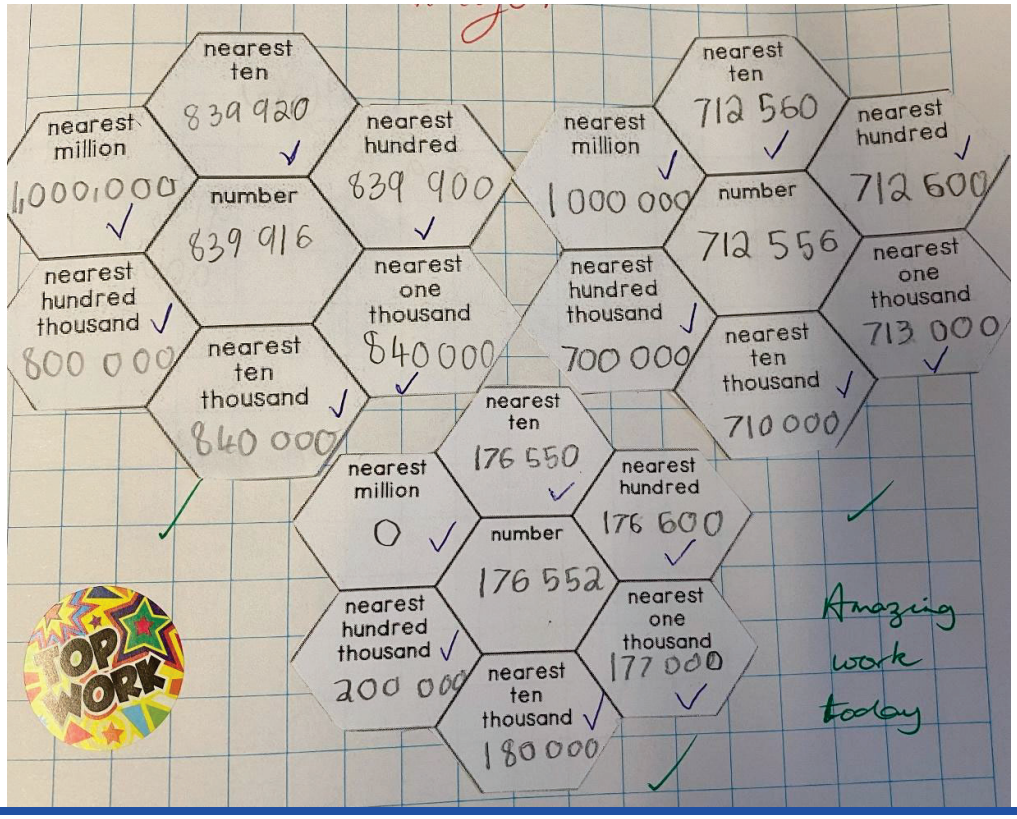
Number lines sliced and at the ready to support the rounding to each parkour base.

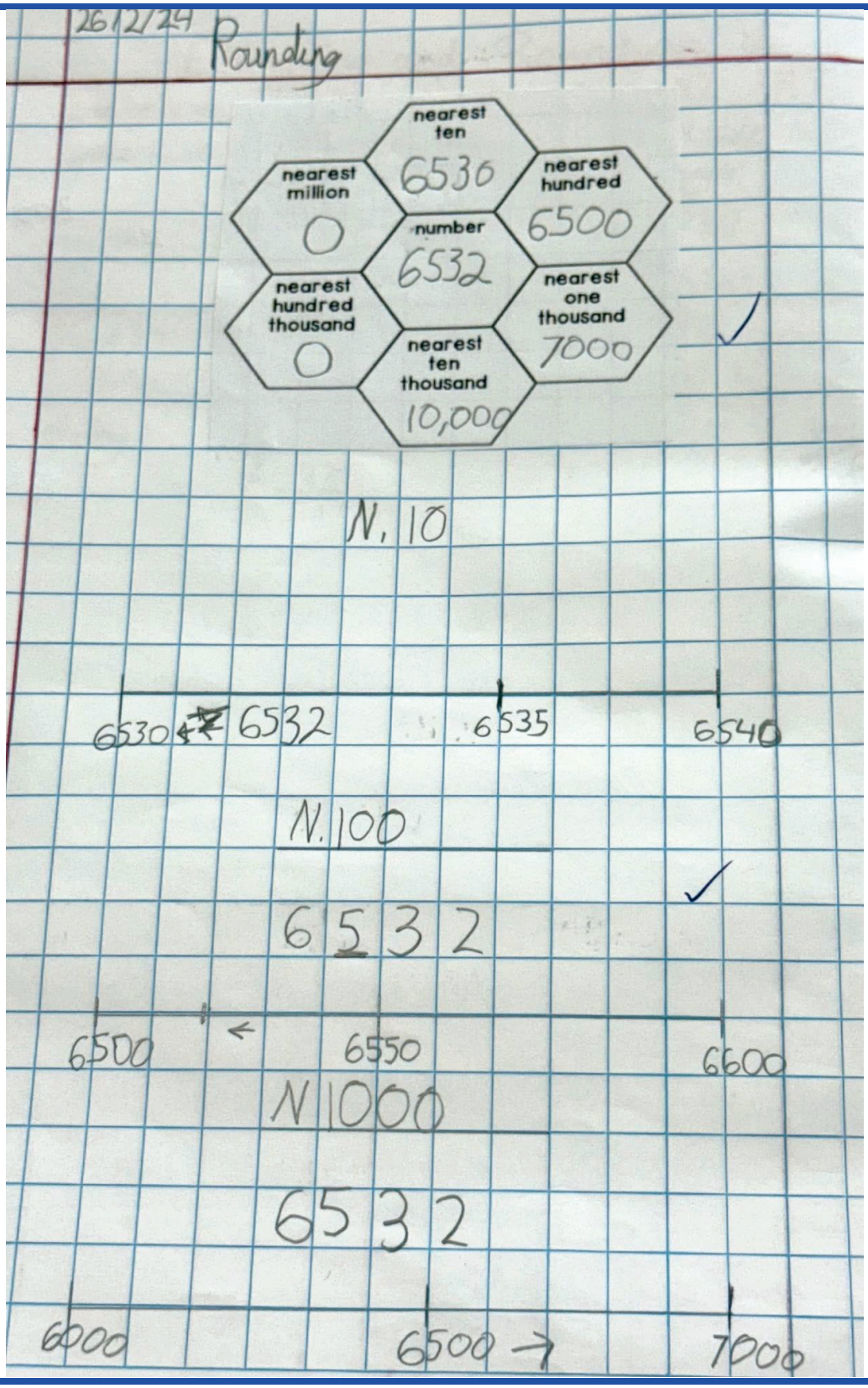


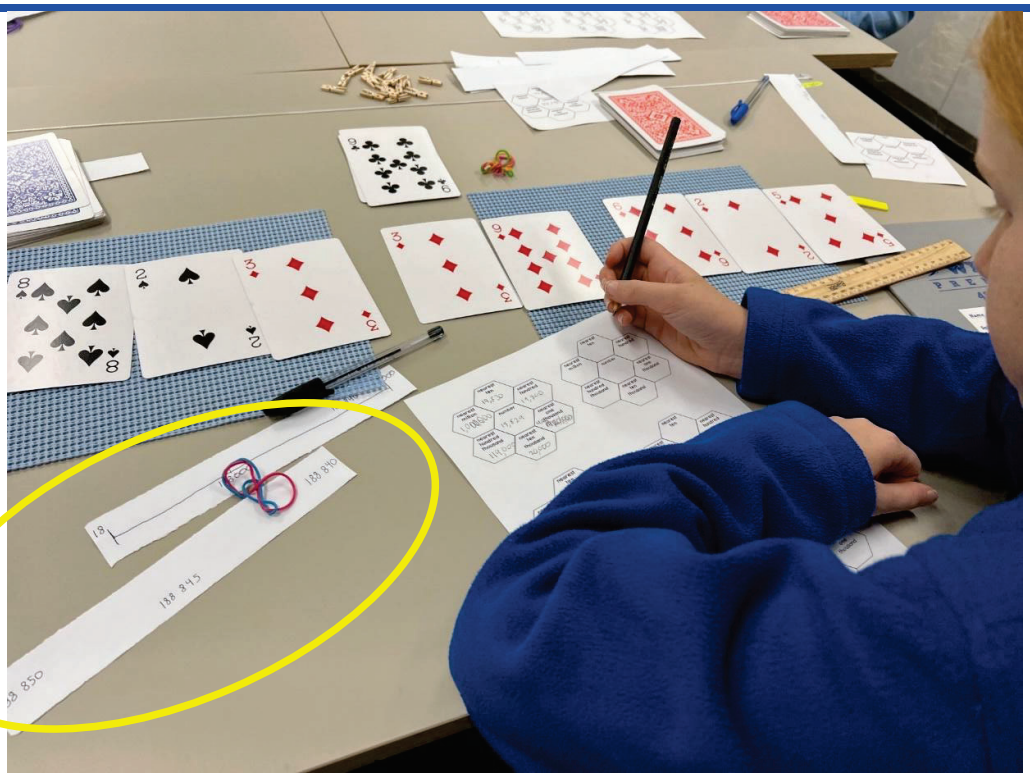
Lesson in action

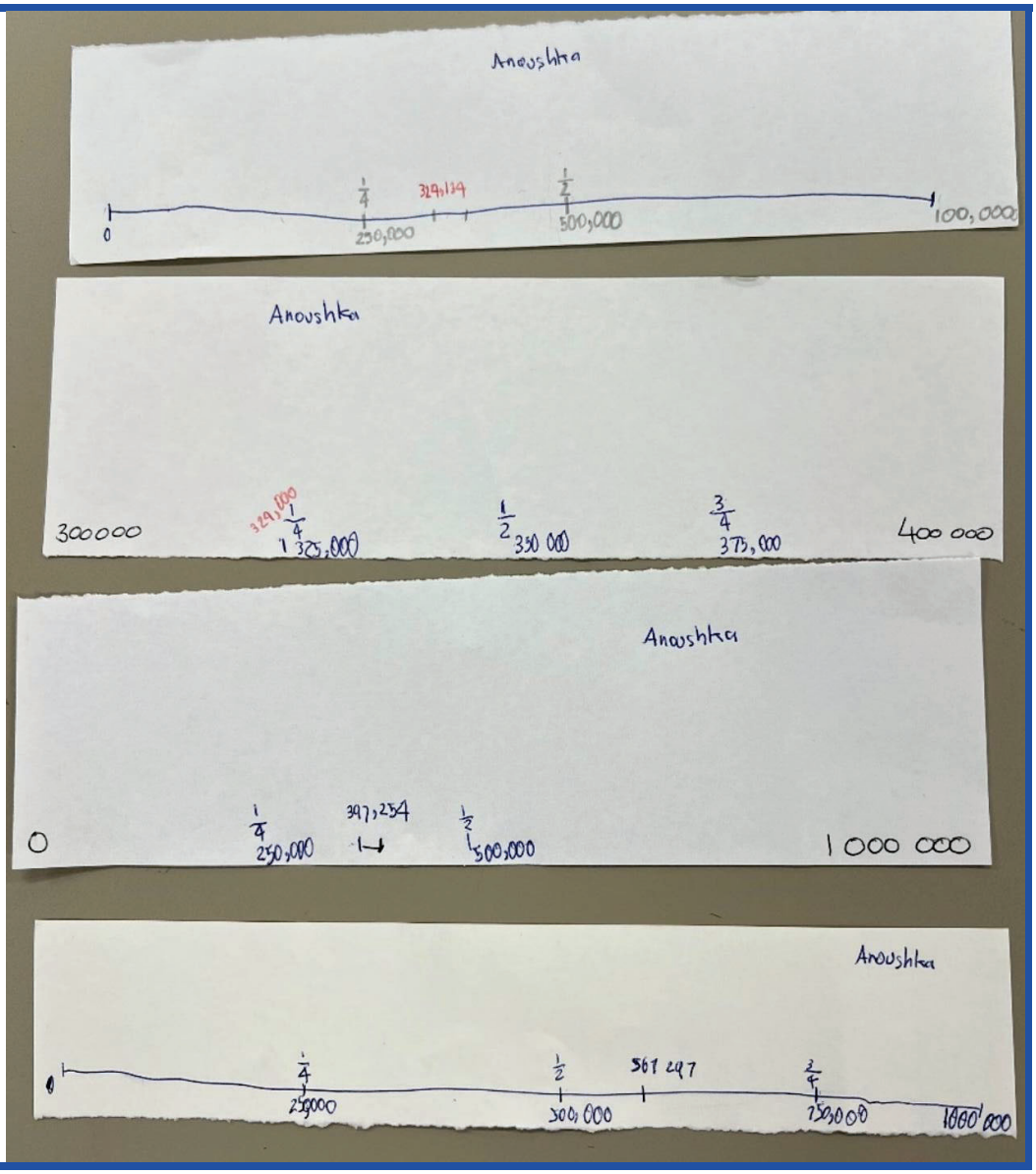


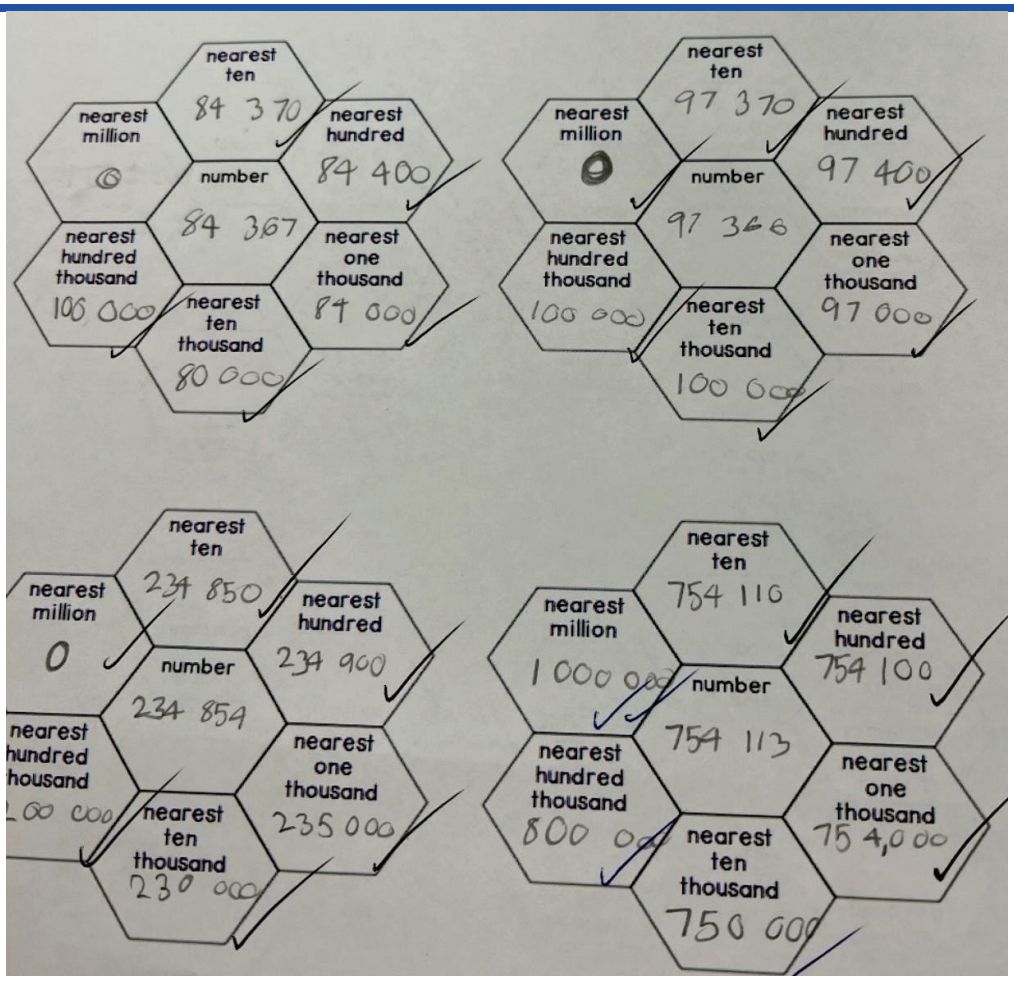


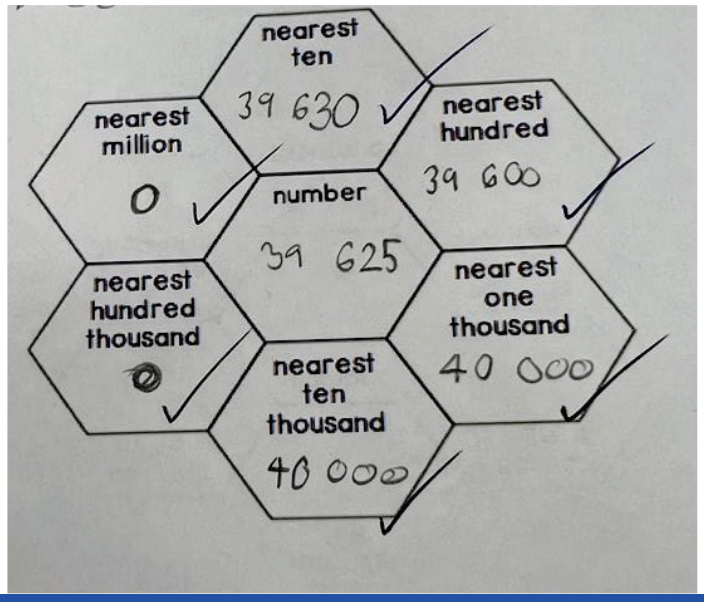




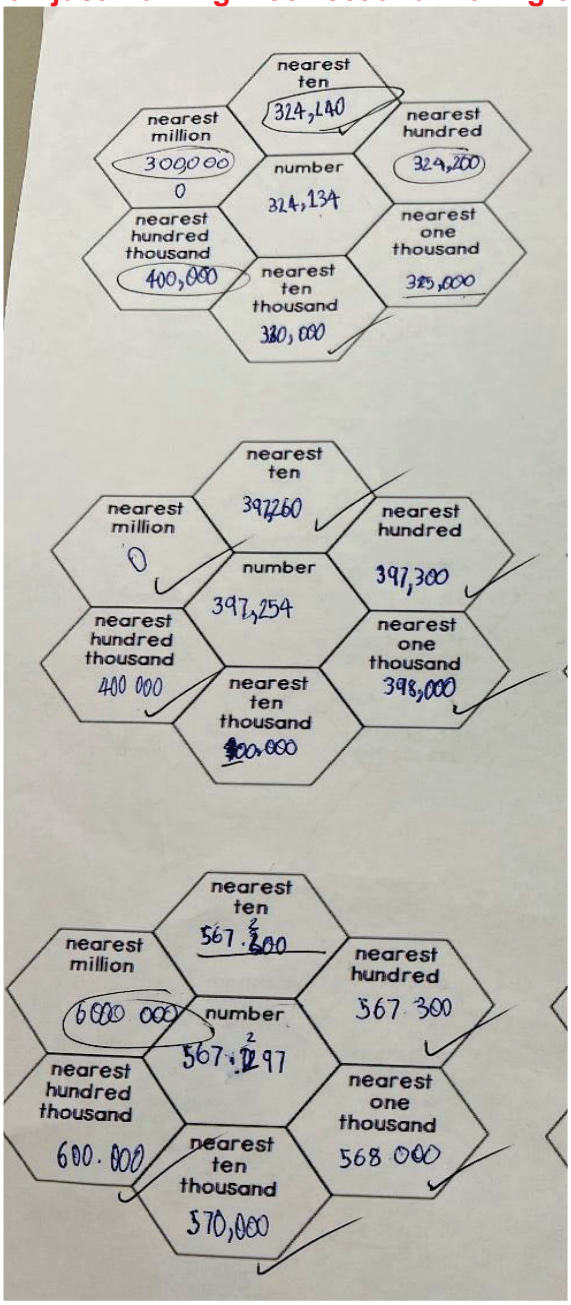






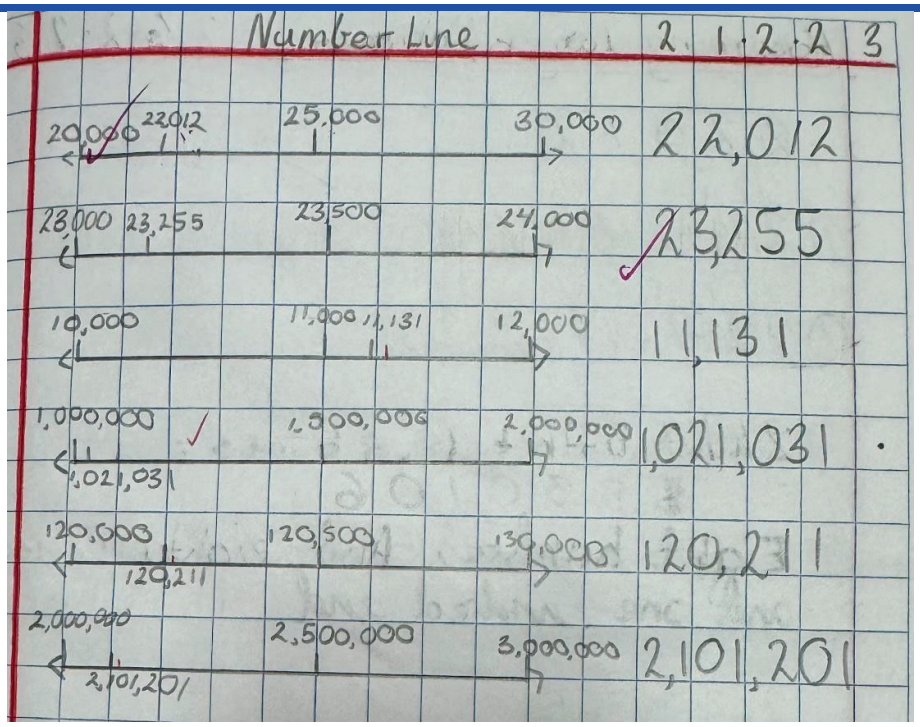


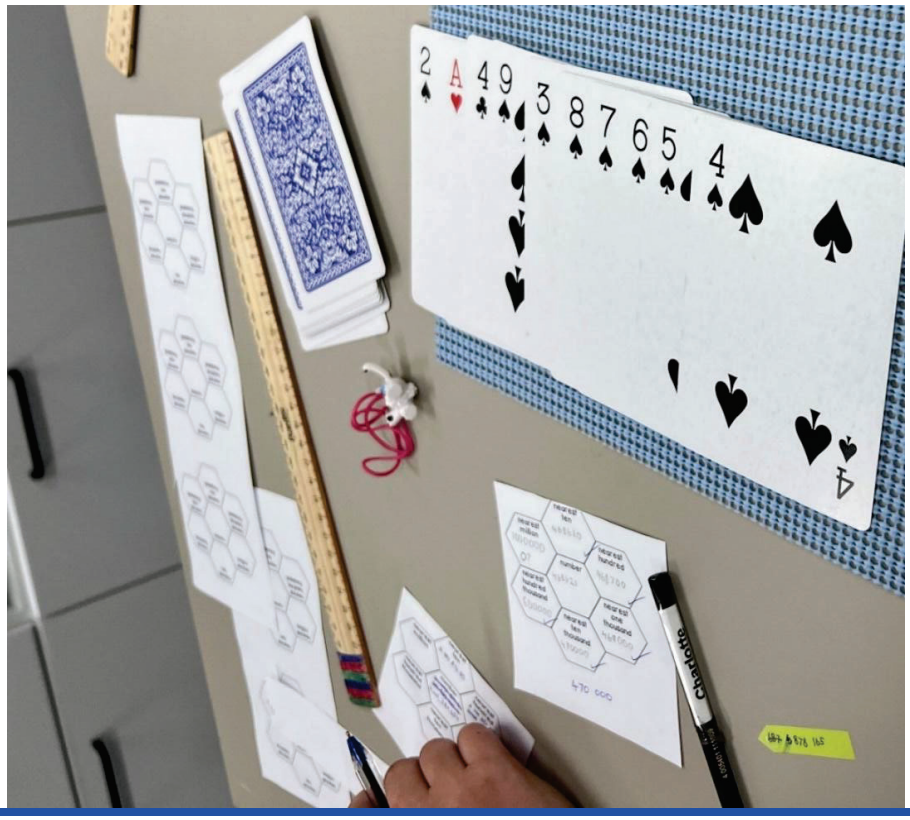
Errors are circled for student to re-attempt at the start of the following lesson (rather than just marking incorrect and moving on):

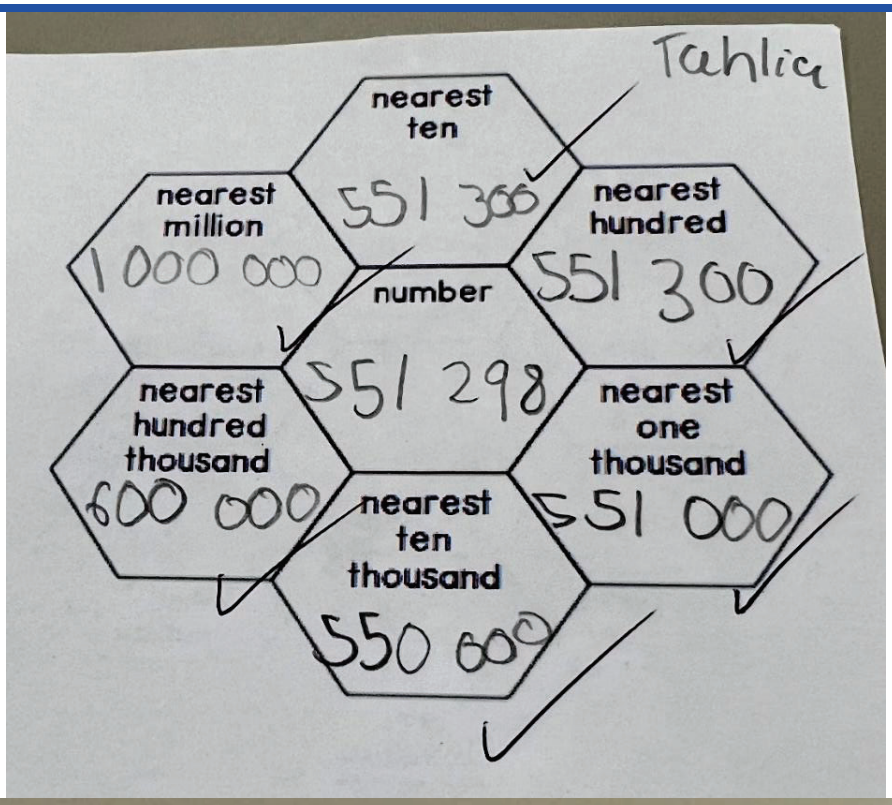


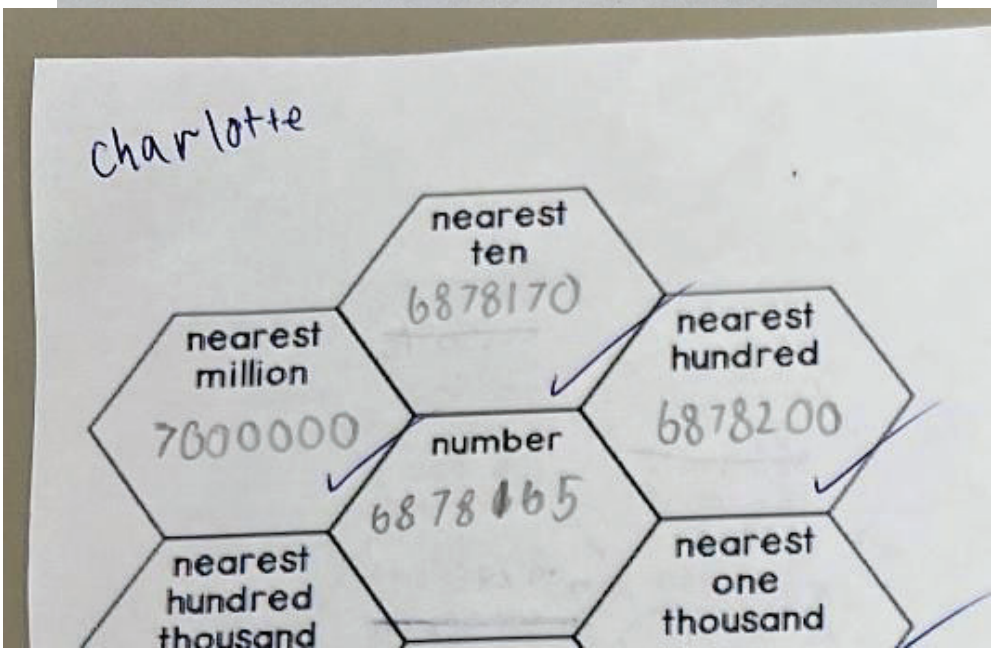
Feedback strategy for repeat sessions – Instead of 'crossing' what students get wrong and correcting it, circle it and ask them to try again. It is critical for students to see and attempt to correct their mistakes, with support obviously, but without the teacher simply telling them the correct answer as feedback. It is far more valuable to circle it and challenge them to re-attempt it, after modelling a similar example together if needed, then provide feedback after the re-attempt.

Lesson in action

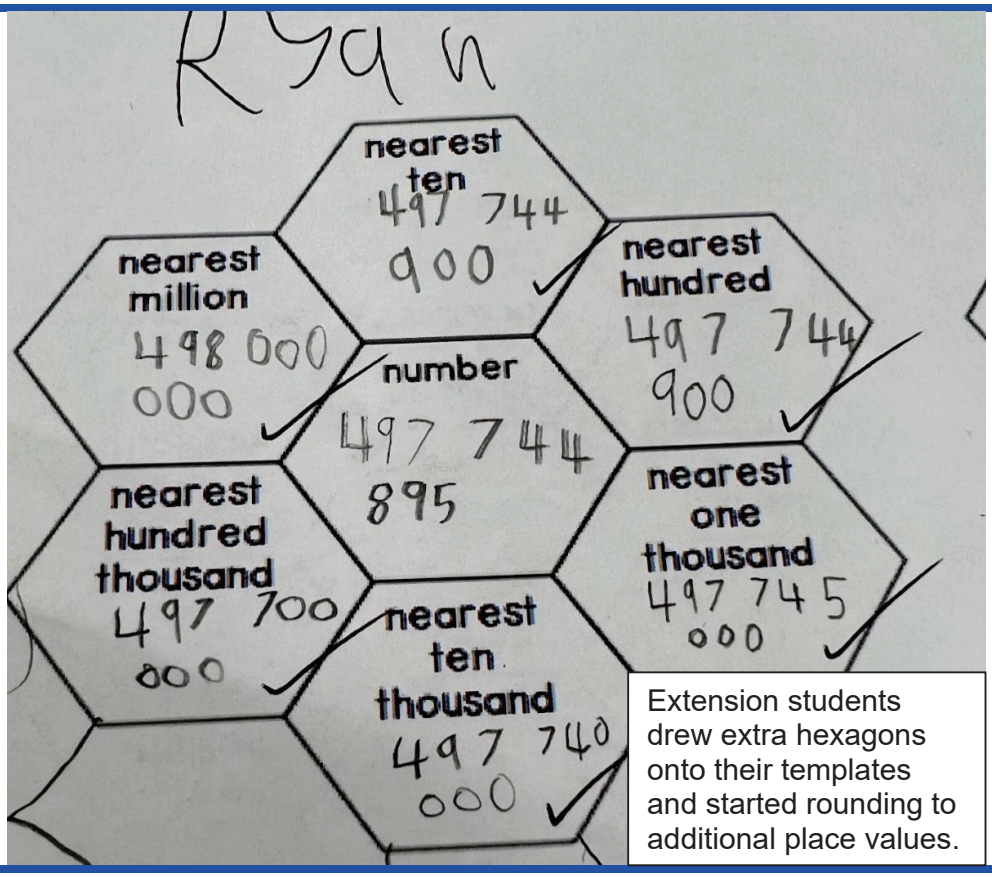




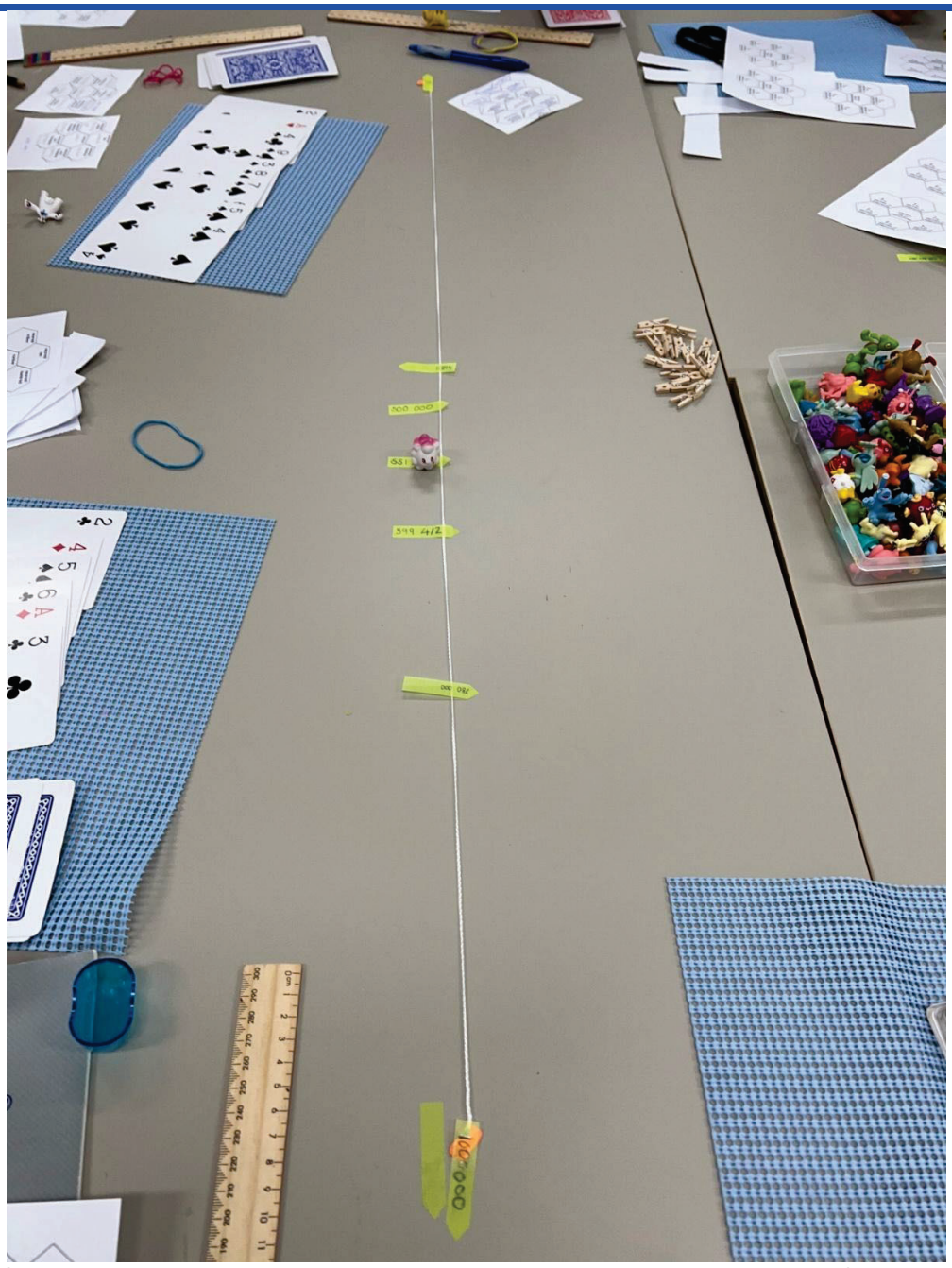




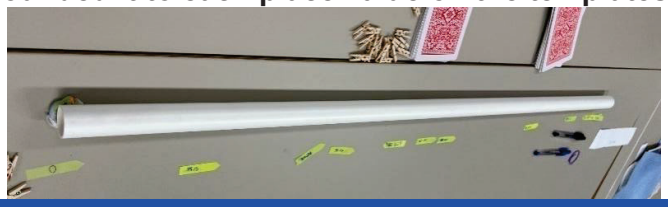
If we compare these two work samples – which is easier to read?
The lower one, as it has <u>spaces between each set of 3 place values</u>.
Model for students to use this strategy.



Extension students drew extra hexagons onto their templates and started rounding to additional place values.



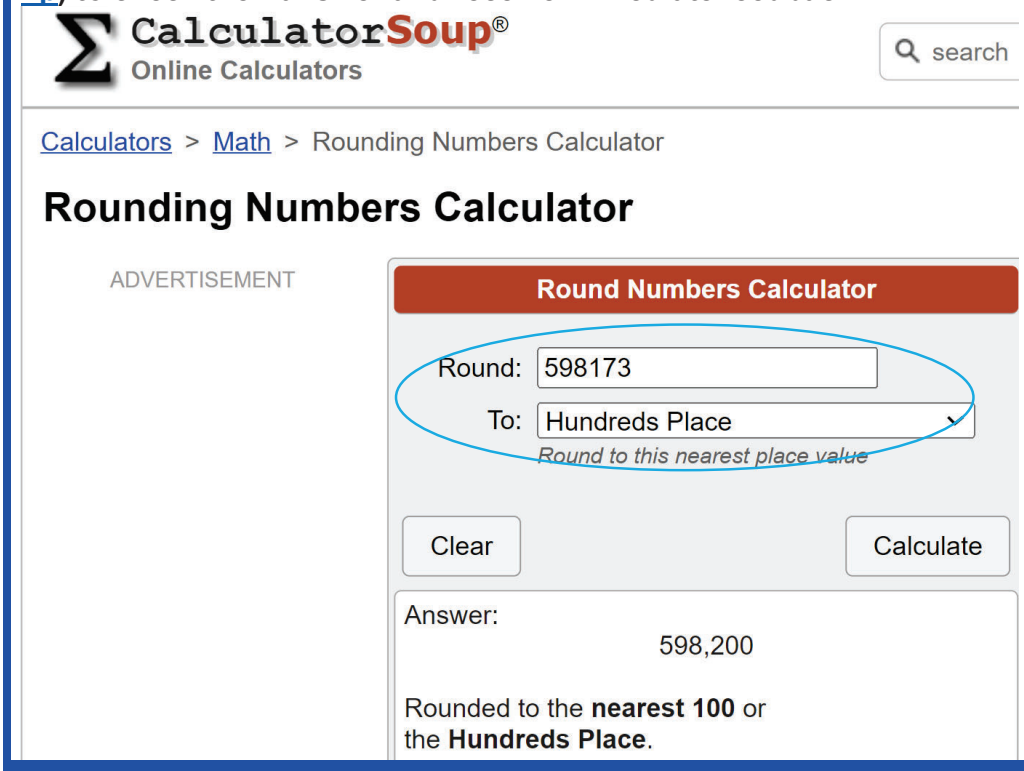
Students added each number to a 0 to 1 000 000 number line after they rounded it to each place value on the templates.



Placing numbers along the line

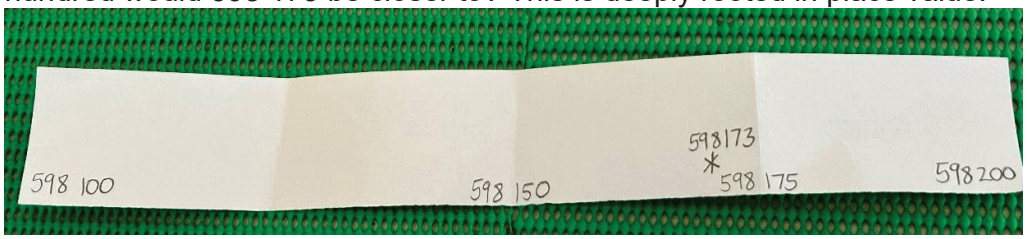
Immediate Feedback

Students can then use this rounding calculator (https://www.calculatorsoup.com/calculators/math/roundingnumbers.php) to check their answer and receive immediate feedback.



Avoiding 'rote/process/digit-focused' teaching

Great strategy: When you round 598 173 to the nearest hundred, think first: What are the two closest hundreds? 100 and 200! So, it will either be 598 100 or 598 200. Finally, which is closer? If you put it on a number line, which hundred would 598 173 be closer to? This is deeply rooted in place value.

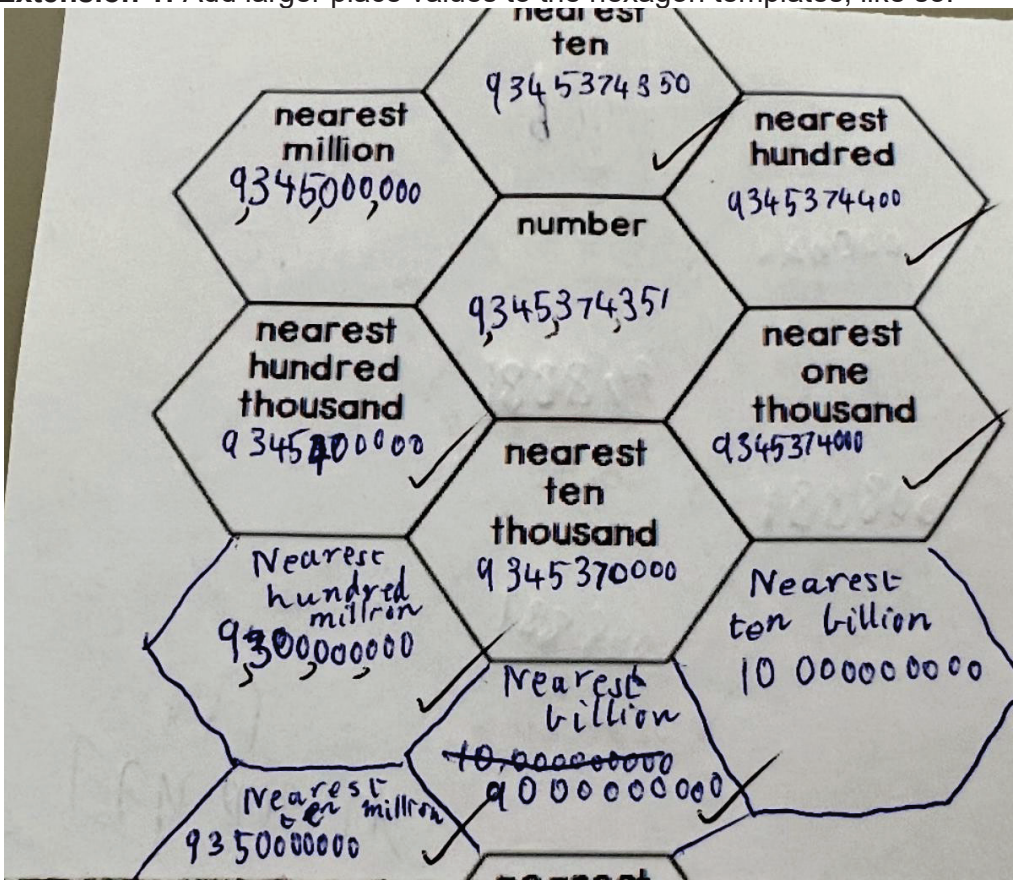


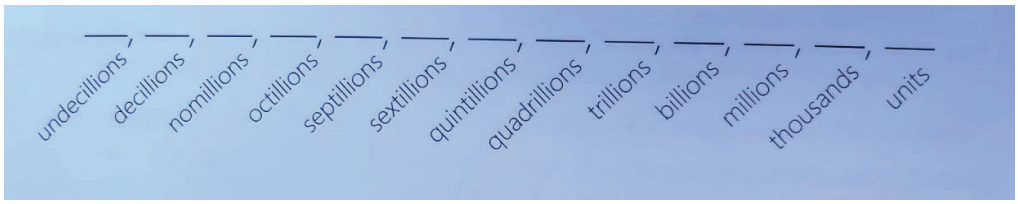
Misguided teaching alert: This is much richer than encouraging students to look at a particular digit, then round up or down. That often leads to an incorrect application of rote-based rules, rather than a place-value based, strategic approach based on a genuine understanding of rounding as 'what is the number closer to.' A 'digit-centric' focus *("4 and below rounds down, 5 and above rounds up")* removes the depth of place value from these skills, creating room for dangerous misconceptions and 'silly' mistakes because students are not using their place value conceptions, but are instead

applying rules they have been told to follow. It then becomes easy to confuse which place needs to be rounded and which digit needs to be focused upon. For this reason, leave the 'rounding mountain' or 'rounding rollercoaster' posters in the cupboard until the very last part of this learning intention – or, better yet, encourage students to create their own versions of anchor charts to summarise the understanding that they formed themselves based on place-value, number-line centric questioning and strategies.

Support: Pull less cards to lower the starting numbers, and only focus on the first parts of the template from the top clockwise (rounding to the nearest ten and hundred), crossing out other parts of the template until they feel ready to attempt these.

Extension 1: Add larger place values to the hexagon templates, like so:





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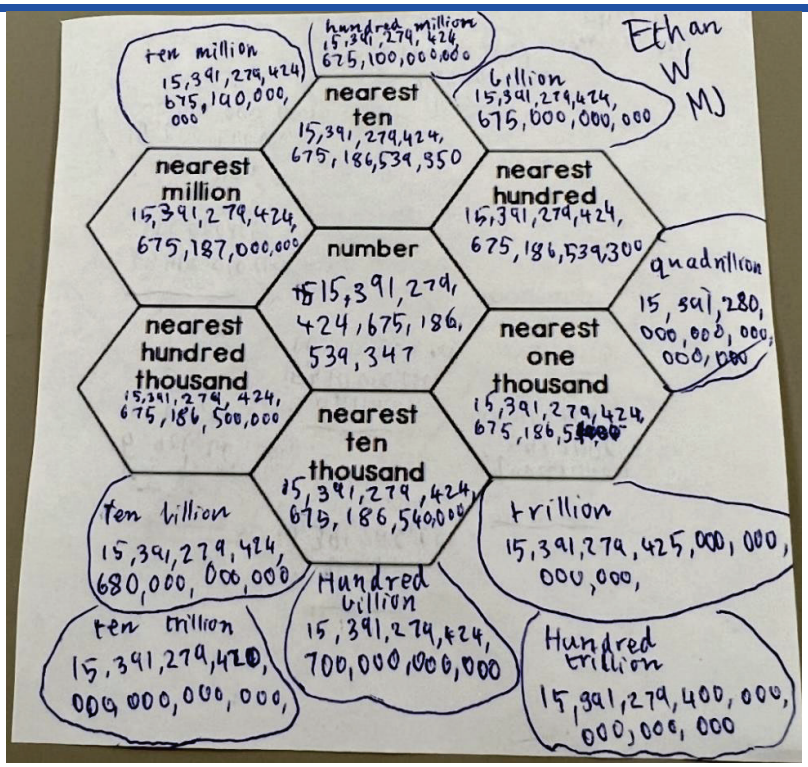
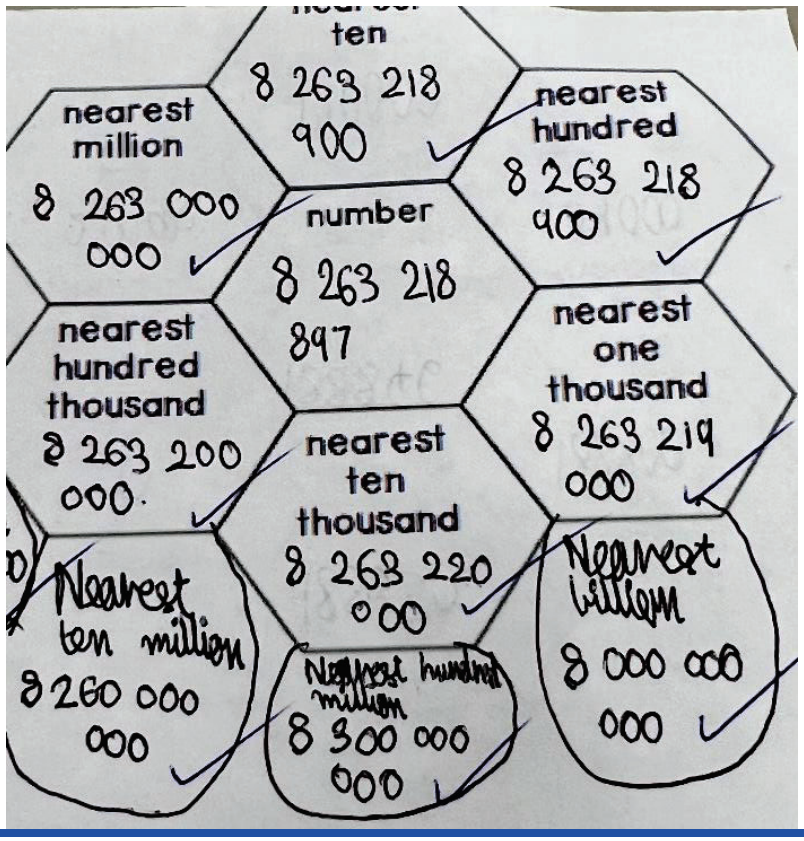
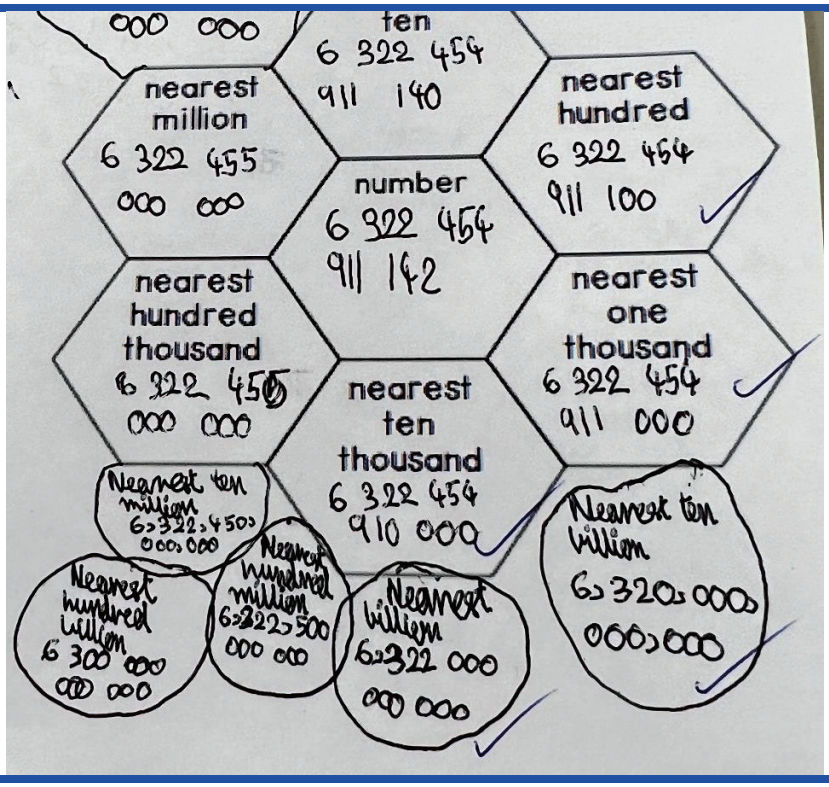
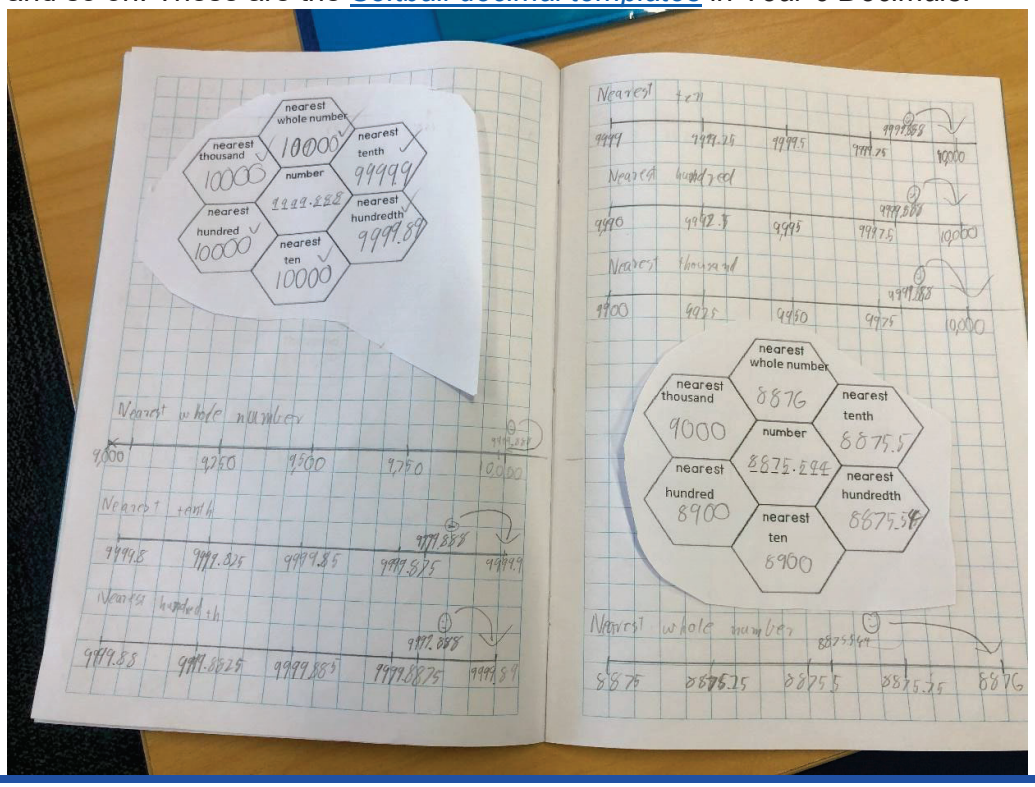
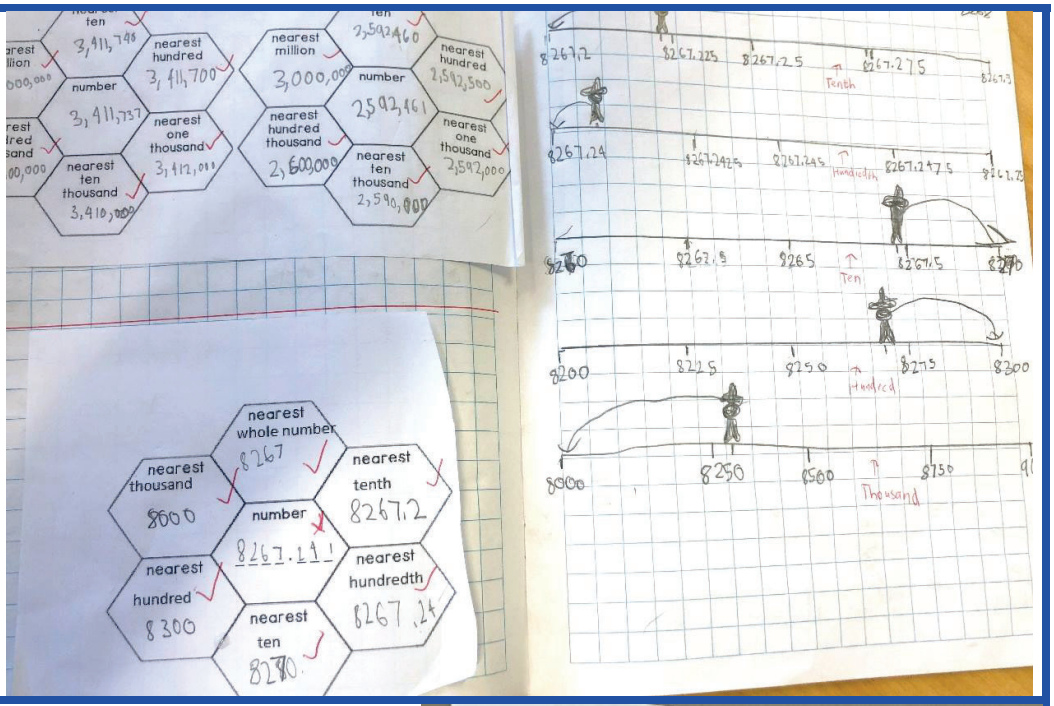






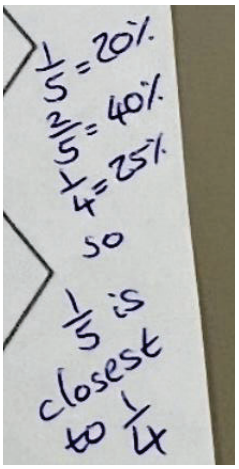
Extension 2: Decimal version – rounding to the nearest tenth, hundredth and so on. These are the *Softball decimal templates* in *Year 6 Decimals*.

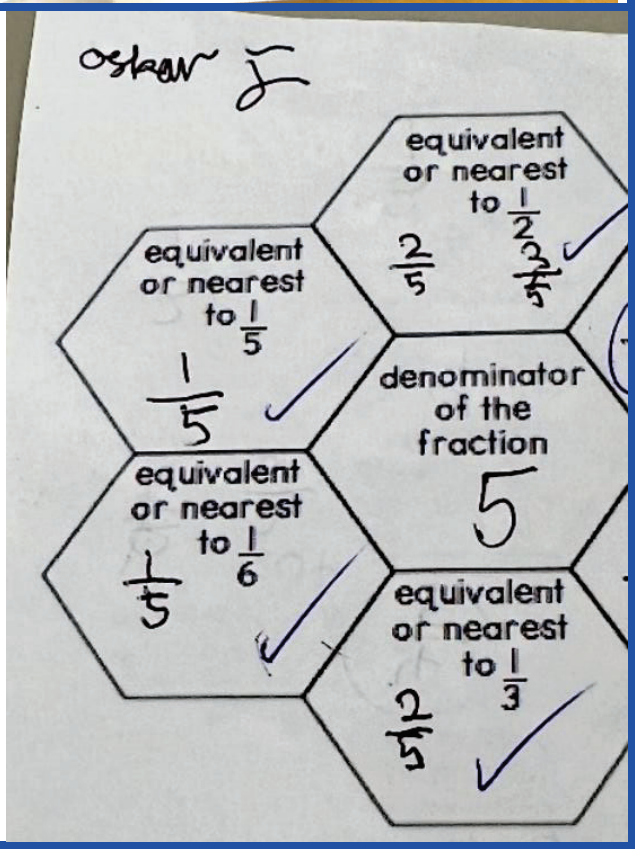


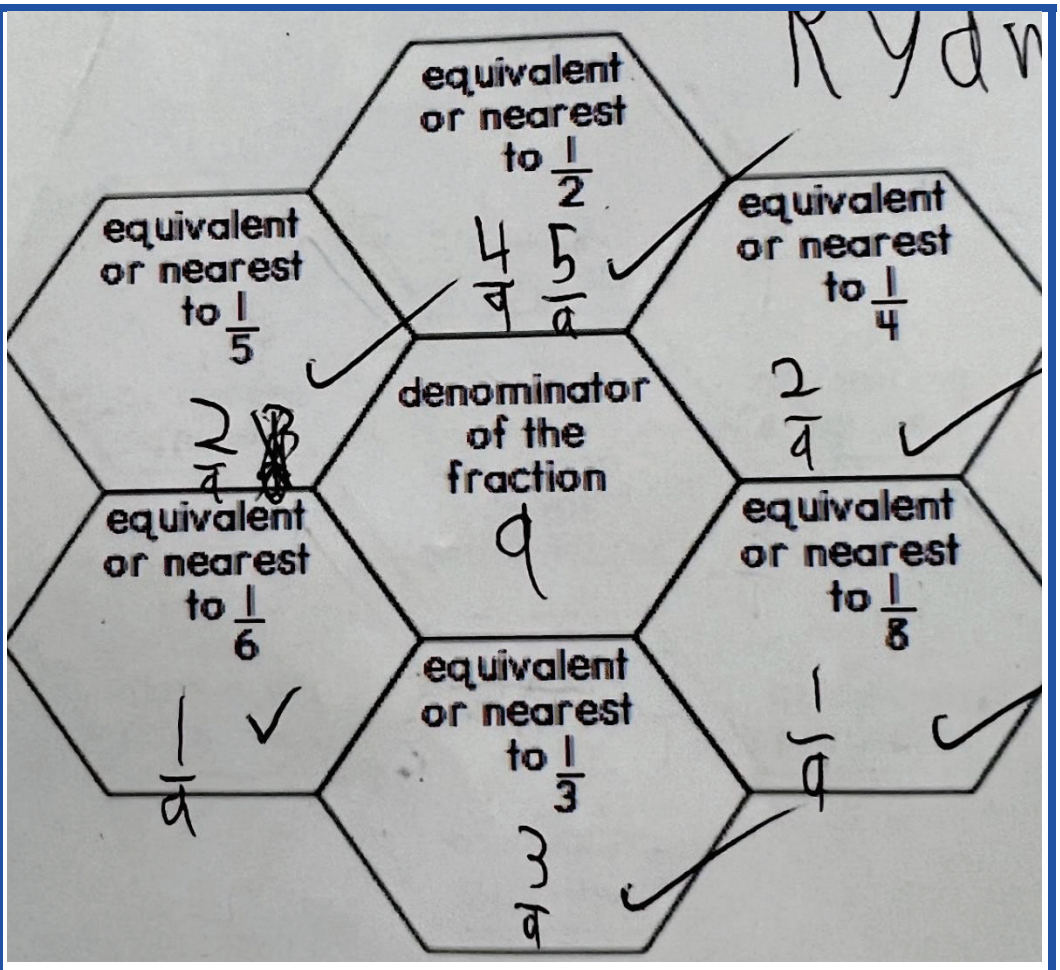


Extension 3: Use the [rounding parkour extension templates](). Pull one playing card as the denominator for a fraction. Then complete the rounding or equivalence to each fraction around the template.

Example reasoning:







If extension students still find this easy, pull two playing cards for the denominator, so that they need to find the closest fractions to all of the above hexagons for a denominator like 62. For example, that $\frac{31}{62}$ is the equivalent fraction to half, or that $\frac{21}{62}$ is closest to one third.

Extension 3: Imagine that the numbers on the playing cards are negative, for example, -238 902. All number lines are then negative. If you are rounding to the nearest ten, which number goes on the left-hand side: -238 900 or -238 910? Negative -238 900 goes on the left, as it is closer to zero and zero always belong on the left-hand side of a number line (in the same way as a measuring tape or ruler).

