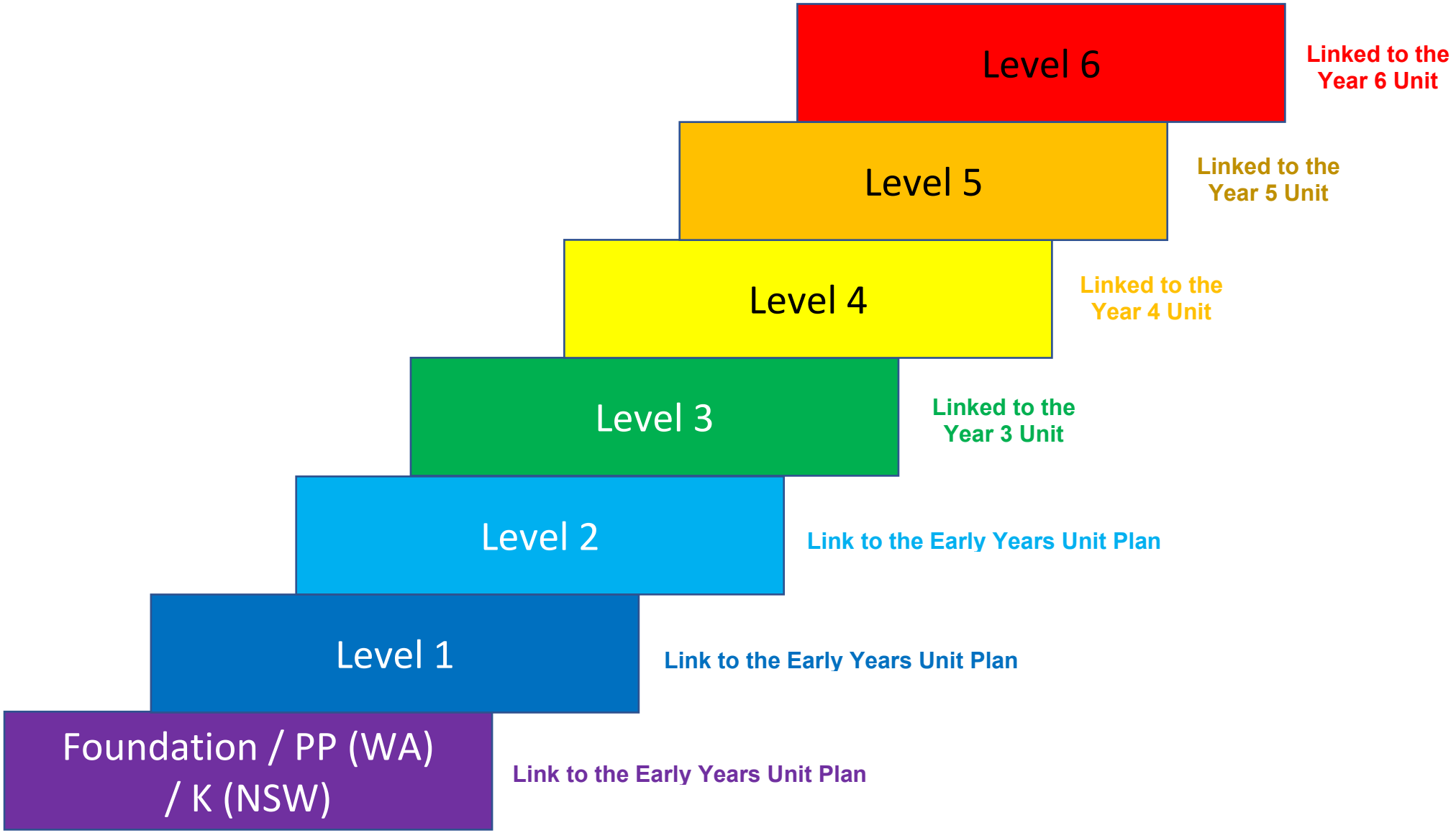
Colour coded by the rainbow



Critical Learning Goals and Checkpoints

Created and used by teams in Top Ten Mathematics schools (www.toptenmaths.com) to provide a strong sequence of big ideas for each year level, and to guide teachers as to the main priorities for their year level for number and algebra.

Some skills intentionally front-load content that can be achieved earlier, such as students learning the names of the tens by skip-counting by 10 up to 120 during the first year of school, even though the curriculum does not require this to be assessed until Year 1.

Skills and Strategies Staircases

Mathematical development follows a sequential staircase of skills and strategies. This means that if one step in the staircase is missing, this will become a major obstacle for the student, unless that skill or strategy is addressed, before progressing to more advanced content.

Consider the build to ten addition strategy as an example. To use this strategy, a student must be able to work out that 8 + 5 can be solved by thinking 8 + 2 (makes 10) + 3 more makes 13. This appears simple for adults who are fluent, but for students this requires the ability to partition 5 fluently into parts, then hold these parts while applying the 10 facts with fluency as well. It also relies partly on the counting on concept, in that students must start from the larger of the two addends (the 8, rather than the 5). In all, students need to have mastered three previous steps in the addition skills and strategies staircase, in order to have any opportunity to master the build to ten strategy. Without ensuring students have already mastered all these previous steps first, a teacher could spend weeks trying to develop their students' understanding and fluency of the build to ten strategy, to no avail.

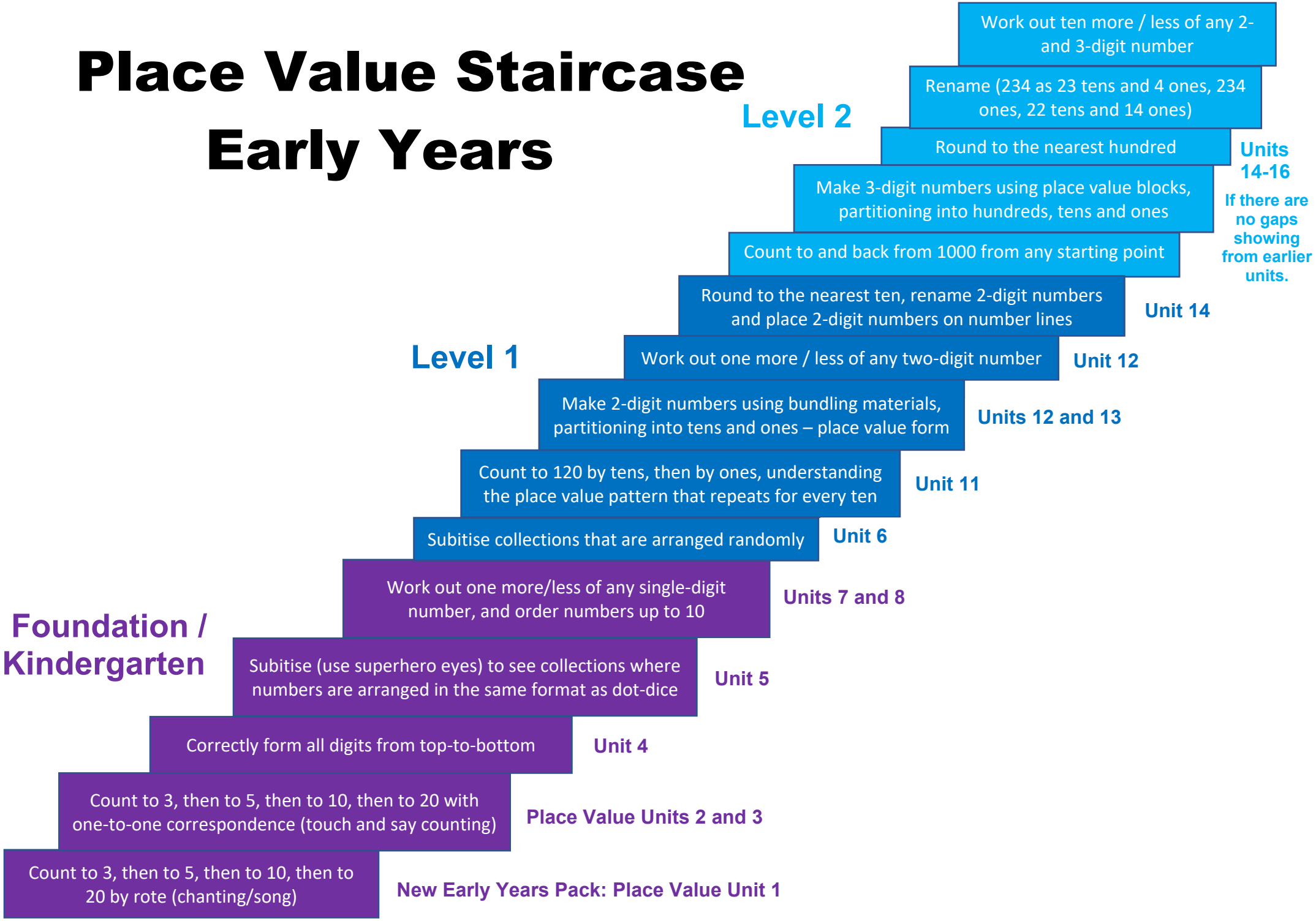
Critical Checkpoints for Place Value

The big goals, by the end of the unit, are that all students in this year level can:

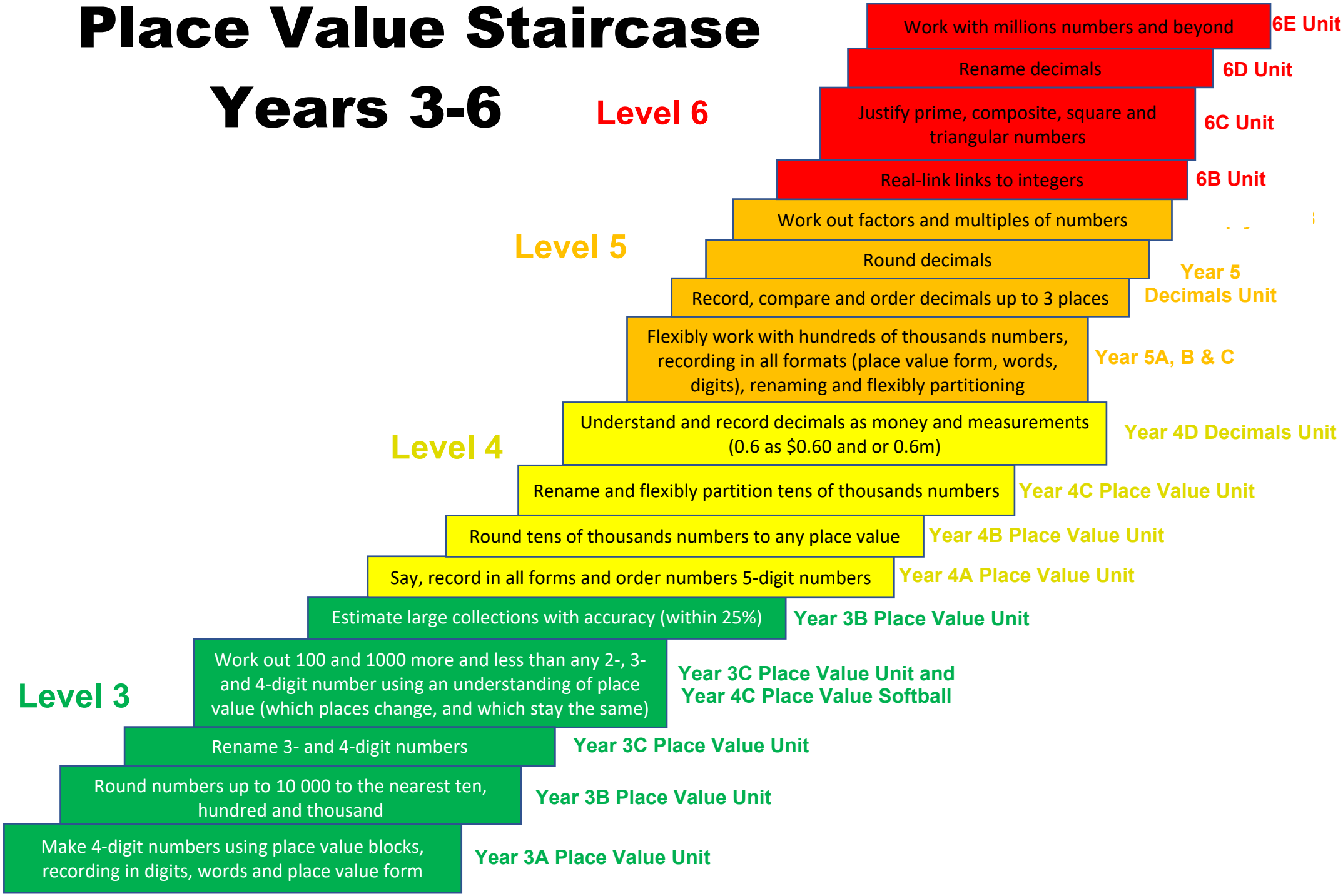
First Year of School	**Count, make and read numbers up to 10** with 1-1 correspondence, says ordinal numbers up to 10	**Subitise regular/dot dice formats up to 6** "I see 6!" or "I see 3, I see 3, I see 6!"	**Correctly form all digits** (0-9 starting from the top, without reversals) Compare and order single-digit numbers	**Fluent with one more / less of any single-digit number**
Level 1	**Count to and back from 120 from any starting point** learning the names of the tens first; bridging over 100 correctly, e.g. 99, 100, 101, 102 e.g. 90, 100, 110, 120	**Subitise irregular / random formats** seeing totals up to 10 using parts, even when arranged randomly, "I see 5, I see 3, I see 8!"	**Make tens (2-digit) numbers** using bundling materials and place value blocks. Record in place value form (34 as 3t 4 ones), standard (34) and worded forms (thirty-four). Rename (3t 4u, 2t 14u). Round to ten.	**Work out one more and one less of a two-digit number**
Level 2	**Count to and back from 1000 from any starting point by ones, tens and hundreds**	**Say, record and order 3-digit numbers** Place value form (452 = 4h 5t 2), standard (452) and worded forms (four hundred and fifty-two).	**Make 3-digit numbers with place value blocks** Round to nearest hundred/ten, place on number lines.	**Work out ten more / less of 2- and 3-digit numbers** Notice patterns (only tens change)

Level 3	**Rename 3- and 4-digit numbers** 340 as 34 tens, 340 ones, 2h + 14t 4530 as 45 hundreds and 3 tens, or 453 tens	**Say, record and order larger places** Place value form (1056 = 1uth 0h 5t 6u), standard and worded forms, focus on internal zeroes.	**Make larger numbers using place value blocks** Round to nearest thousand/ hundred/ ten, number lines.	**Work out 100 and 1000 more/less of any 4-digit number**
	Estimate collections accurately and round to many place values			
Level 4	**Connect decimals to money and measurement**	**Say, record and order 5-digit numbers** Including place value form	**Flexibly work with 5-digit numbers** number lines, round to all places, rename	**Recognise equivalence in fractions**
Level 5	**Work out the factors and multiples of 2-digit numbers** Connect factors to divisibility (if you divide 63 by 7, there will be no remainders, since 7 is a factor of 63)	**Flexibly work with 6-digit numbers** Round to all places; rename; place on number lines; flexible expanded form (168 350 + 150 000, partition 168 350 into 150 000 and 18 350, and double the 150 000 first)	**Compare and order decimals up to 3 places** (decimats, measurement)	**Round decimals**
Level 6	**Make real-life links to integers and place on number lines**	**Justify why a number is prime, composite, square or triangular**	**Rename decimals**	**Work with millions and beyond**

Place Value Staircase
Early Years



Place Value Staircase
Years 3-6



Critical Checkpoints for Addition

The big goals, by the end of the unit, are that all students in this year level can:

First Year of School	**Count all to solve an addition situation and retell it orally using the word 'and',** e.g. "I have 3 jelly beans AND you have 4 jelly beans. We have 7 jelly beans altogether!"	**Orally partition numbers up to 6** (can name all the combinations/ways to make 3, 4, 5 and 6 without materials, e.g. 3 and 3 makes 6, 4 and 2 makes 6, 5 and 1 makes 6)	**Fluent with one more than any single-digit number** e.g. 7 + 1, immediately thinks 7 and 1 more is 8 (without materials), later 2 more
Level 1	**Draw or write an addition (and) story to match an equation** e.g. the student is given 3 + 4, they draw, write or retell orally: "I have 3 red M&Ms and 4 blue M&Ms, I have 7 altogether!"	**Orally recall the 10 facts and partition all numbers up to 10** (knows all the ways to make 7, 8, 9 and 10)	**Count on from the larger number by using turnarounds (commutativity) for single-digit additions** e.g. 2 + 5, start at 5 and counts 2 more mentally
Level 2	**Fluently recall the doubles facts up to 10 + 10** (2+2, 3+3, 4+4, 5+5, 6+6, 7+7, 8+8, 9+9, 10+10)	**Use near doubles and explain how they did it** (6 and 7, I thought 7 and 7 – 1 OR 6 and 6 + 1 more)	**Build to 10 and explains how** (e.g. 7 and 5, I did 7 + 3 + 2; or, alternatively, I thought 6 + 6 by moving 1 from the 7 to the 5)
	Add ten to any 2-digit and 3-digit number Notices the pattern that only the tens change, ones stay the same		

Level 3	**Add 5 or more numbers using the best strategies for that problem** (10s facts, doubles, near doubles, building to 10)	**Split, jump and switch strategies up to 3-digits <u>mentally</u>** Estimate by rounding first	**Mentally solve ways to make 100** (62 *and what* makes 100?)
Level 4	**Estimate answers by rounding** Always estimate before solving	**Split, jump and switch strategies up to 4 digits <u>mentally</u>**	**Mentally solve ways to make 1000** (329 *and what* makes 1000?) Calculate change to the nearest 5 cents
Level 5	**Estimate decimal addition by rounding**	**Selectively choose strategies and use a second to double check**	**Use the vertical algorithm for larger additions** (<u>only</u> when split, jump and switch are not efficient)
	Add and subtract fractions with the same or related denominators		
Level 6	**Choose the best strategy to solve multi-step worded problems involving addition** (split, jump, switch, vertical; excluding irrelevant information)		**Use all addition strategies to add decimals** Estimate first by rounding to the nearest whole
	Add and subtract fractions with different denominators		

Addition Skills and Strategies Staircase

Critical Checkpoints for Subtraction

The big goals, by the end of the unit, are that all students in this year level can:

First Year of School	**Model a subtraction with materials and retell it orally using 'take away'** (e.g. "I had 3 jellybeans, but my brother took one away. 2 were left!")	**Use fingers and drawings to solve subtraction problems**	**Solve one less than any single-digit number** e.g. 7 – 1, immediately thinks 1 less than 7 is 6 and can solve without materials
Level 1	**Fluently recall the backwards 10 facts** (10 – 3 = 7, because I know 7 + 3 = 10) **Solve a simple worded subtraction problem**	**Count back to solve subtractions from up to 20**	**Count on to solve the difference between numbers to 10** (e.g. 9 – 7, "8 9, it's 2")
Level 2	**Use backwards doubles and explain how** (e.g. 14 – 7, I thought 7 + 7 = 14, so 14 – 7 = 7) **Create a simple worded subtraction problem**	**Counts and jumps back to solve 2-digit subtractions,** including jumping back by tens, e.g. 80 – 17, start with 80 – 10 to get to 70, then 70 – 7 = 63	**Can count on to solve the difference between numbers up to 20** (e.g. 20 – 17, the difference is 3)

Level 3	**Apply fact families to solve subtraction** (e.g. 108 – 102, I know 102 + 6 is 108, so it is 6)	**Jump back, jump the difference and transformation strategies for 3-digit problems**	**Mentally solve ways to make 100** (38 *and what* makes 100?)
Level 4	**Estimate answers by rounding** Always estimate before solving	**Jump back, jump the difference and transformation strategies for 4-digit problems**	**Mentally solve ways to make 1000** (671 *and what* makes 1000?) Calculate change to the nearest 5 cents
Level 5	**Estimate decimal subtraction by rounding**	**Selectively choose strategies and use a second to double check**	**Use the vertical algorithm for larger subtraction** (only when mental strategies are not efficient)
	Add and subtract fractions with the same or related denominators		
Level 6	**Chooses the best strategies to solve complex worded subtraction problems** (jump back strategy, jump the difference strategy, vertical algorithm, get to 9 strategy)		**Use the vertical algorithm to subtract decimals**
	Add and subtract fractions with different denominators		

Subtraction Skills and Strategies Staircase

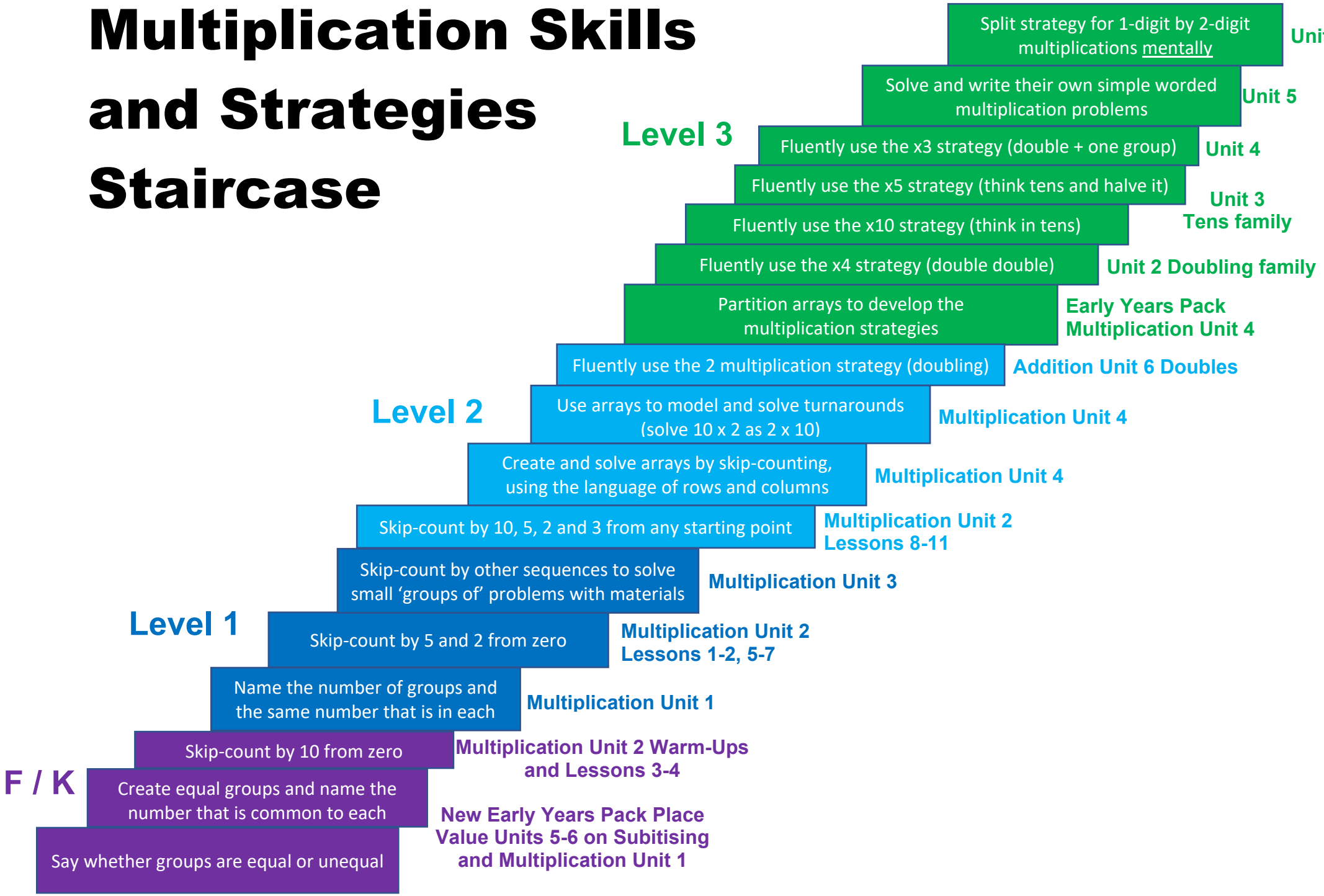
Critical Checkpoints for Multiplication

The big goals, by the end of the unit, are that all students in this year level can:

First Year of School	**Say whether groups are equal (fair) or unequal (unfair)**	**Create equal groups using objects, naming the number that is common to each**	**Skip-count by 10 from zero up to 120** (learn the names of the tens)
Level 1	**Name how many groups there are and the same number that is in each group**	**Use skip-counting as a strategy to solve small totals, recording using repeated addition number sentences**	**Skip-count by 5 and 2 from zero**
Level 2	**Create arrays and solve these by skip-counting** (by 10, 2, 5, 3 and 4); use arrays to model turnarounds (3 x 2 = 2 x 3)	**Fluently know the 2 (doubling) multiplication strategy** knows the multiples and factors, can write fact families for the 2 times tables	**Skip-count by 10, 5, 2 and 3 from any starting point**
Level 3	**Partition arrays to develop multiplicative strategies** (3x6 as double 6 and 6 more, 2x18 as double 18, 4x6 as double double 6)	**Use the doubling strategy to solve x4 (double double)** **Use double + group to solve x3**	**Use place value to solve x10** **Solve x5 as x10 halve it, or halve it x10**

Level 3	**Write and solve 'groups of' problems given a set equation** **(e.g. create a real-life problem about 10 x 3, swap with a partner and solve)**		
	Split strategy (4 x 23, think 4 x 20 and 4 x 3 = 80 + 12 = 92)		
Level 4	**Multiply 2-digit numbers by applying strategies** **4 x 23 think double double**	**Use the doubling strategy to solve x8 (double double double)**	**Use the tens family to solve x9 (x10 take away one group)**
	Solve x6 using x5 + group, or x3 double it strategy, or memory	**Solve x7 using x5 + double, or another strategy, or memory**	**Use the area model for 1-digit by 3-digit problems**
	Apply multiplicative thinking to solve real-life problems		
Level 5	**Estimate answers using rounding and place value patterns** **(e.g. 17 x 52, round to 20 x 50** **2 x 5 = 10, 2 x 50 = 100** **So: 20 x 50 = 1000) E ≈ 1000**	**Uses the area model to multiply 1-digit by 4-digits, and 2-digits by 2-digits**	**Use lattice <u>or</u> vertical algorithm to solve 2-digits by 2-digits**
	Use factors strategies (double one factor, halve the other)		
Level 6	**Use their <u>preferred strategy</u> (lattice, area, vertical) to solve 2-digit by 3-digit multiplication**	**Choose their preferred strategy to solve multi-step worded problems**	**Multiply decimals by powers of ten**

Multiplication Skills and Strategies Staircase

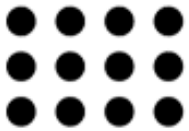


Multiplication Skills and Strategies Staircase

Critical Checkpoints for Division

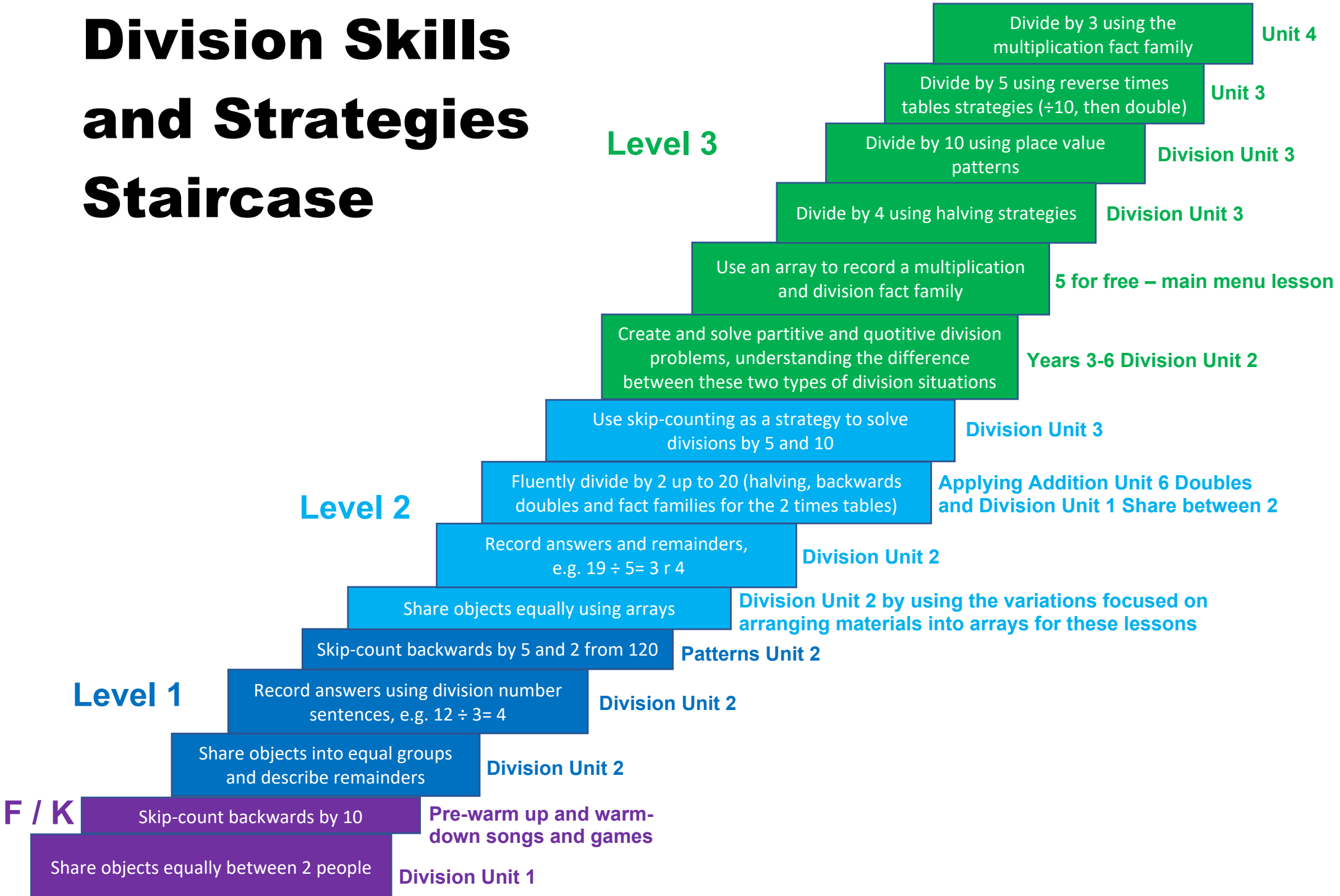
The big goals, by the end of the unit, are that all students in this year level can:

First Year of School	**Share objects equally between 2 people**		**Skip-count backwards by 10 from 120**
Level 1	**Share objects into equal groups and describe remainders** e.g. "13 shared between 5 gives 2 to each with 3 remainders"	**Skip-count backwards by 5 and 2 from 120**	**Write the number sentence for shared between situations with materials showing** e.g. 12 ÷ 3 = 4
Level 2	**Share objects using arrays** (*arra*nged into equal numbers of rows and columns)	**Fluently divide by 2 (halving),** backwards doubles, fact families for twos **Use skip-counting as a strategy to solve divisions by 5 and 10**	**Write division number sentences that include remainders** e.g. 20 ÷ 7 = 2 r 6
Level 3	**Create and solve both partitive and quotitive problems** (Partition: 8 balloons, 4 people, how many did each person get? Quotition: 8 balloons, each person got 2, how many people?)	**Fluently divide by 10 and 5 using the tens family** (to divide by 5, divide by 10 then double it) **and divide by 4 using half half**	**Fluently divide by 3 using multiplication fact families**

<table>
<tr><td>Level 3</td><td colspan="3">Using an array, record the full multiplication and division fact family
 
3 rows of 4 is 12 — $3 \times 4 = 12$
4 columns of 3 is 12 — $4 \times 3 = 12$
12 shared into 3 rows is 4 — $12 \div 3 = 4$
12 shared into 4 columns is 3 — $12 \div 4 = 3$</td></tr>
<tr><td rowspan="3">Level 4</td><td colspan="2">Use fact families to solve and justify answers to divisions by 6, 7 and 9
(49 ÷ 7 = 7 because 7 x 7 = 49)</td><td>Fluently use halving strategies to divide
(to divide by 8 think half half half, as well as dividing larger numbers by 2 and 4 using repeated halving)</td></tr>
<tr><td colspan="3">Estimate answers to larger division problems by applying power of ten patterns to single-digit multiplication facts</td></tr>
<tr><td colspan="3">Use the reverse area model to partition a dividend into easier to divide parts</td></tr>
<tr><td rowspan="3">Level 5</td><td colspan="3">Uses near multiplication facts to solve divisions with remainders (27 ÷ 4, thinks 4 x 6 = 24 , so 4 x 6 + 3 = 27, so it is 6 r 3)</td></tr>
<tr><td colspan="3">Apply multiplicative thinking to solve real-life problems</td></tr>
<tr><td>Use the multiply to divide strategy to solve divisions with remainders
253 ÷ 6, solve by:
6 x <u>40</u> = 240, 6 x <u>2</u> = 12, so 42 r 1 = 42 $\frac{1}{6}$
OR 240 ÷ 6 = 40
13 ÷ 6 = 2 r 1, so 42 $\frac{1}{6}$</td><td>Use different notations to represent division
$25 \div 4, 4\overline{)25}, \frac{25}{4}$</td><td>Record remainders as fractions and decimals where appropriate for the context of the problem
$25 \div 4 = 6\frac{1}{4}$ or 6.25</td></tr>
</table>

<table>
<tr><td></td><td colspan="3">Understand and apply the divisibility tests</td></tr>
<tr><td rowspan="2">Level 6</td><td>Divide 4-digit numbers by 1-digit using short division or another preferred strategy</td><td>Use the multiply to divide strategy or short division for 4-digit divisions by a 2-digit number on paper e.g. to solve 564 ÷ 18:
18 x 10 = 180
18 x 20 = 360
18 x 30 = 540
So, it will be 30 groups of 18 + 1 more group of 18 (540 + 18= 558) with 6 remainders, so 31 r 6</td><td>Chooses the best strategies to solve complex worded problems involving division
(estimating, multiply to divide strategy, reverse area model, short division algorithm)</td></tr>
<tr><td colspan="3">Divide decimals by powers of ten</td></tr>
</table>

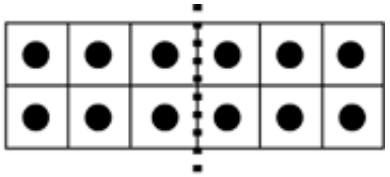
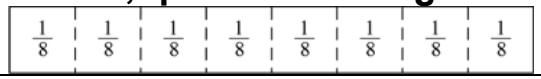
Division Skills and Strategies Staircase

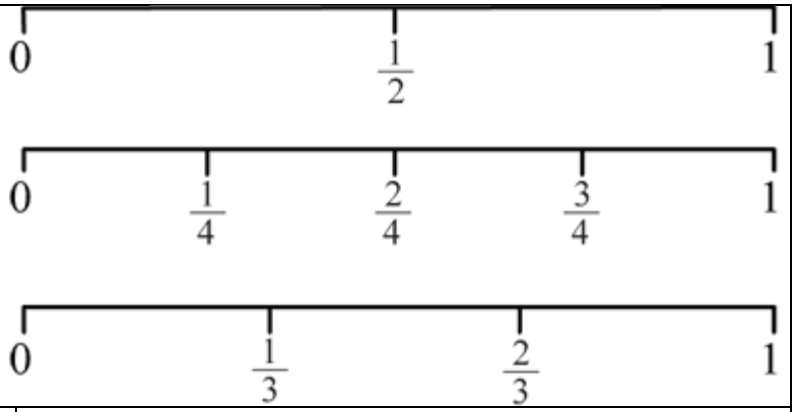
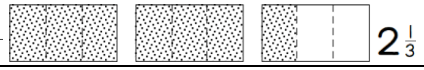


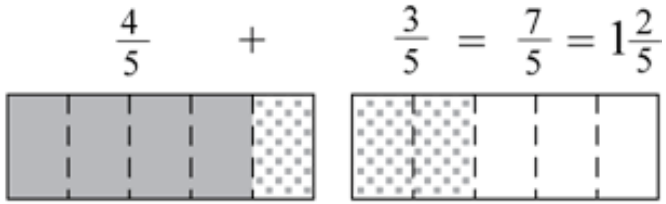
Division Skills and Strategies Staircase

Critical Checkpoints for Fractions and Decimals

The big goals, by the end of the unit, are that all students in this year level can:

<table>
<tr><td>First Year of school</td><td colspan="3">Use ‘out of’ language e.g. 2 out of 3 of the students at the front are girls
Know their ordinal numbers to at least 10</td></tr>
<tr><td>Level 1
<u>(not required by Aus V9 or Vic 2.0, but front-loading for Level 2 content nevertheless)</u></td><td>Describe two equal parts of a whole as two halves
explains when something is not half and why</td><td>Use fraction notation for halves, understanding this as 1 <u>out of</u> 2 parts
 
  </td><td>Solve half of collections
  </td></tr>
<tr><td>Level 2</td><td>Create halves, quarters and eighths
Strategy: Cut in half, then half (for quarters), then half again (for eighths)
Use fraction notation for halves, quarters and eighths
  </td><td>Solve quarters and eighths of collections
Strategy: Split the whole collection in half, then half (for quarters), then half again (for eighths)</td><td>Compare halves, quarters and eighths,
e.g. $\frac{1}{8}$ is less than $\frac{1}{4}$ because it is 1 <u>out of</u> 8 parts, instead of 1 <u>out of</u> 4
e.g. $\frac{1}{2}$ is more than $\frac{1}{4}$ because it is 1 out of 2 parts, rather than 1 out of 4 parts</td></tr>
<tr><td>Level 3</td><td>Real-life representations of thirds, fifths and tenths (as well as halves, quarters and eighths)
with any numerator</td><td>Recognise equivalence to one whole, 5 out of 5 parts makes one whole, so five fifths are needed to make one whole
rename 1 as $\frac{2}{2}$ $\frac{3}{3}$ $\frac{4}{4}$ $\frac{5}{5}$ $\frac{8}{8}$</td><td>Count by halves, thirds, quarters, eighths, fifths and tenths up to one whole</td></tr>
</table>

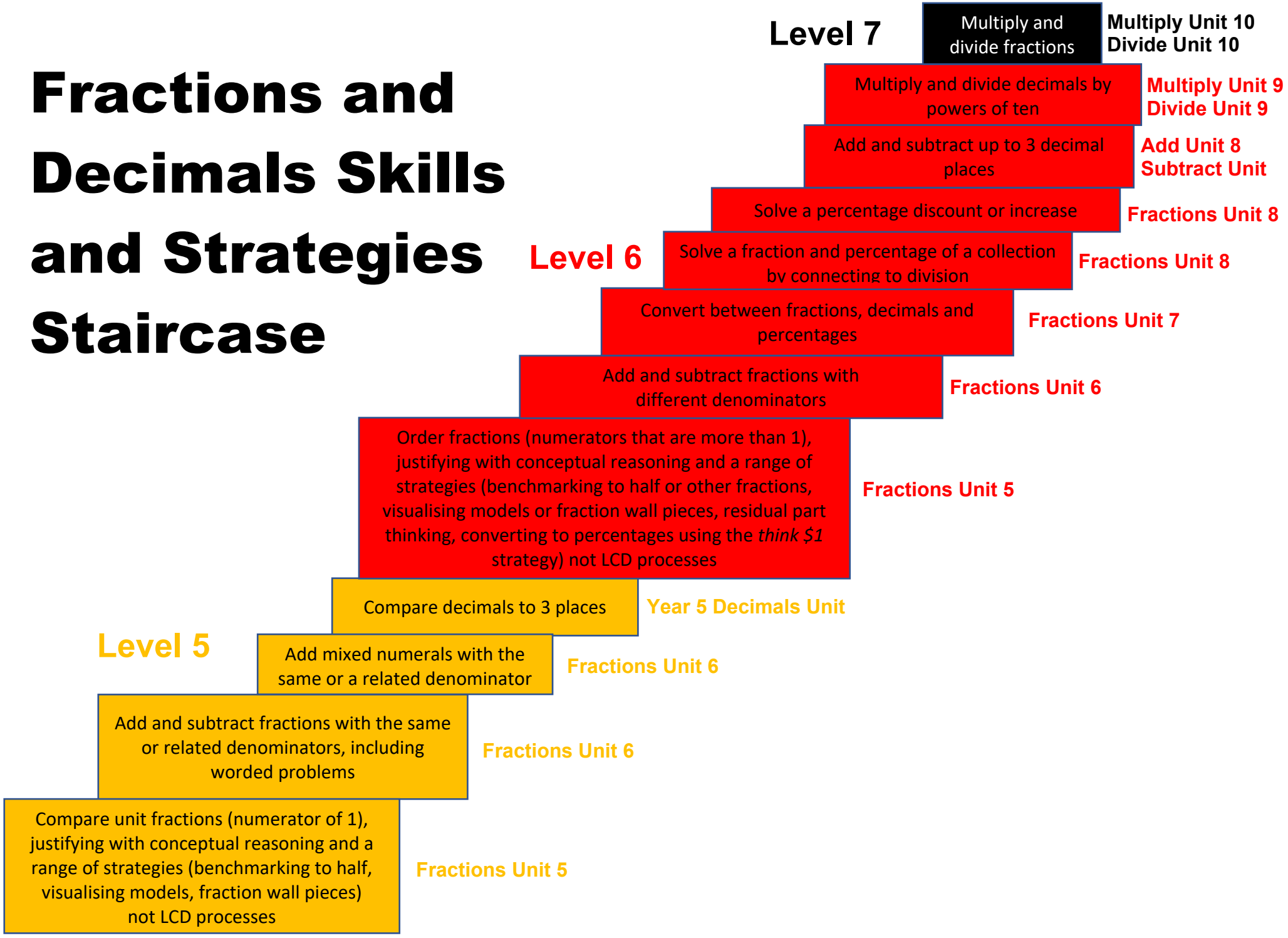
<table>
<tr><td>Level 3</td><td colspan="2">Place halves, quarters, eighths, thirds, fifths and tenths on parallel number lines</td><td> 
</td></tr>
<tr><td rowspan="3">Level 4 Fractions</td><td colspan="2">Understand, identify and create equivalent fractions</td><td>Connect fractions to tenths and hundredths Understand that $\frac{1}{10}$ is the same as 0.1 because both show 1 out of 10 parts (1 out of 10 parts of $1)
<b>$\frac{1}{100}$ is the same as 0.01 because both show 1 out of 100 parts (1 out of 100 parts of $1)</td></tr>
<tr><td colspan="3">Convert between improper and mixed fractions, and count by fractions above one whole
$\frac{7}{3}$  $2\frac{1}{3}$</td></tr>
<tr><td colspan="2">Represent decimals as money
Understanding the wholes as dollars and parts as cents, separated by the decimal point.
Show money in decimal and fraction notation, e.g. $5.35 as 5 whole dollars and 35 out of 100 cents:
$5\frac{35}{100}$ or 5 wholes, 3 ten cents (tenths) and 5 single cents (hundredths): $5+\frac{3}{10}+\frac{5}{100}$</td><td>Represent decimals as measurement
Understanding wholes as metres and parts as centimetres, separated by the decimal point
50cm = 0.50m = 0.5m
140cm = 1.40m = 1.4m (zeroes at the end do not change the value)</td></tr>
<tr><td>Level 4 Decimals</td><td colspan="3"></td></tr>
</table>

<table>
<tr><td rowspan="2">Level 5</td><td>Compare unit fractions (numerators of 1), justify the comparison with conceptual <u>reasoning</u> (not LCD)</td><td colspan="2">Compare, order and place decimals of up to 3 places on number lines</td></tr>
<tr><td colspan="2">Add and subtract fractions (including worded problems) with the same denominator, including where the result is an improper fraction or mixed numeral
 
</td><td>Add mixed numerals with the same or a related denominator
$2\frac{1}{5} + 1\frac{2}{5} = 3\frac{3}{5}$</td></tr>
<tr><td>Level 6 Fractions</td><td>Compare, order and place fractions on the same number line (fractions with <u>any</u> numerator) using a wide variety of strategies and justify with conceptual <u>reasoning</u> (not LCD)</td><td colspan="2">Add and subtract fractions with <u>different</u> denominators</td></tr>
</table>

Converting between fractions, decimals and percentages **Level 6 Operating with decimals** **(also mentioned in previous units)**	**Solve a fraction and percentage (10%, 25%, 50%) of a collection, connecting this to division** **e.g. to work out $\frac{3}{8}$ of 40, divide 40 by 8 (half half half) = 5, then multiply by 3 (3 x 5 = 15)** **e.g. to work out 25%, think half of half**	**Convert between fractions, decimals and percentages** '25% means 25 out of 100 or $\frac{1}{4}$ or 0.25' 75%, 0.75, $\frac{3}{4}$ $1.37 = 137\% = \frac{137}{100} = 1\frac{37}{100}$
	Add or subtract decimals to 3 places **First estimating the answer by rounding**	**Multiply and divide decimals by powers of ten**

Fractions and Decimals Skills and Strategies Staircase

Fractions and Decimals Skills and Strategies Staircase



Critical Checkpoints for Patterns and Algebra

Note: **Many of these checkpoints overlap with *Multiplication*.**

The big goals, by the end of the unit, are that all students in this year level can:

<table>
<tr><td rowspan="3">First Year of School</td><td>Sort objects in different ways and describe the categories they used to classify each collection</td><td colspan="2">Continue and create AB (red, blue, red blue) and similar patterns with objects (shapes, counters), sounds (claps), actions (stomps)</td></tr>
<tr><td colspan="3">Understand that a pattern must repeat and be predictable</td></tr>
<tr><td colspan="3">Skip-count by 10 from zero (in advance of curriculum)</td></tr>
<tr><td rowspan="2">Level 1</td><td>Continue and create ABC, ABB, ABBC patterns or similar with objects, sounds and actions</td><td>Solve patterns counting by ones up to and back from 120</td><td>Skip-count by 5 up to 120 and backwards</td></tr>
<tr><td>Skip-count by 2 up to 120 and backwards</td><td colspan="2">Continue, create and describe rules for patterns that increase or decrease by 10, 5 or 2</td></tr>
<tr><td>Level 2</td><td colspan="3">Understand, explain and continue additive patterns that increase or decrease by a constant amount 3, 7, 11, __, 19, 23, 27</td></tr>
<tr><td rowspan="2">Level 3</td><td colspan="3">Understand the equal sign (=) means “is the same as” and must balance in value on both sides</td></tr>
<tr><td colspan="2">Solving missing parts in single-digit addition and subtraction number sentences to balance equations</td><td>Skip-count by 3 and 4</td></tr>
</table>

<table>
<tr><td rowspan="2">Level 4</td><td>Solve missing parts in 2-digit addition and subtraction problems

$\square + 55 = 83, \square - 15 = 19$</td><td>Skip-count by 6, 7, 8 and 9 using partitioning and ten more compensation strategies
(to skip-count by 8, use a ten more, 2 back strategy, particularly for non-zero starts)</td></tr>
<tr><td colspan="2">Continue, create and describe rules for patterns that increase or decrease by fractions, including mixed numbers and improper fractions</td></tr>
<tr><td>Level 5</td><td colspan="2">Solving missing parts in multiplication or division number sentences to balance equations</td></tr>
<tr><td rowspan="3">Level 6</td><td colspan="2">Apply the order of operations and understand its use in real-life</td></tr>
<tr><td colspan="2">Use a table of values for geometric and number patterns to record and solve values of a much later term in the same pattern

e.g. How many matches will you need to make 100 squares?

□ , □□ , □□□ , □□□□ , … ⇨
<table>
<tr><td>number of squares</td><td>1</td><td>2</td><td>3</td><td>4</td><td>…</td><td>100</td></tr>
<tr><td>number of matches</td><td>4</td><td>8</td><td>12</td><td>16</td><td>…</td><td>□</td></tr>
</table>

The pattern is multiplying by 4, so you will need 400 matches to make 100 squares</td></tr>
<tr><td colspan="2">Create and solve input/output tables involving multiple steps</td></tr>
</table>