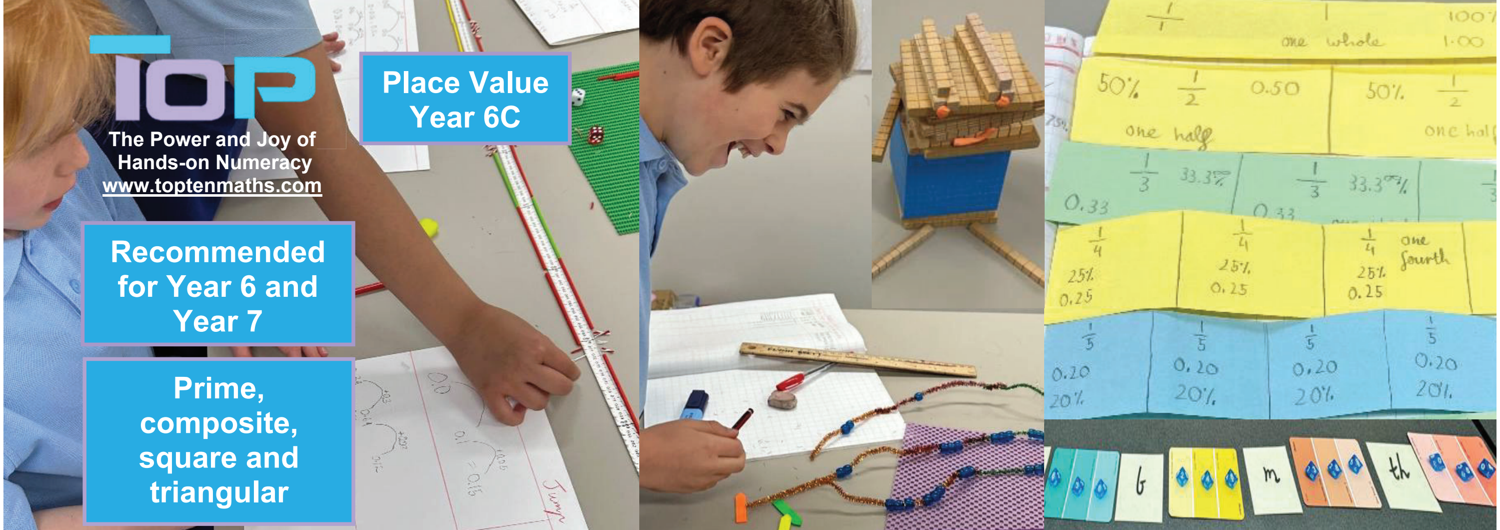

TOP

The Power and Joy of Hands-on Numeracy

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Place Value Year 6C

Recommended for Year 6 and Year 7

Prime, composite, square and triangular

Real-Life Numeracy Years 3-6

Planning Package

Sequential units with hands-on, real-life numeracy for Year 3, Year 4, Year 5 and Year 6 students

Ten years of development in Australian classrooms.

Genuinely high engagement and conceptual understanding in middle to upper primary numeracy.

Comprehensive differentiation for wide ranges: Pre-planned and workable enabling and extending prompts for every lesson.

High-impact, high-relevance professional learning on a daily basis to support planning.

Comprehensive diagnostic and formative assessments to target each sequential point-of-need.

Please note: It is not intended for teachers to attempt to deliver every lesson in this sequence, nor read the unit in full.

Units are designed as **a menu of options**, depending on the points-of-need for each class, with enabling and extending prompts included for every lesson.

Please choose lesson options based on assessed points-of-need (units are directly linked to the assessments), using either Top Ten's or other **strategy-focused diagnostic pre-assessments.** We recommend avoiding multiple-choice/click-the-answer tests, as numeracy as a discipline grows students' reasoning and thinking skills, ability to explain and show strategies, as well as deep conceptual understanding. Answers alone are not the ultimate goal, or a worthy aspiration in the absence of student reasoning.

Please also select lessons that best suit students' interests and your own creativity and passion. Units are designed to share the wisdom of practice, while respecting and safeguarding the professional role of the teacher as the ultimate best judge of students' needs.

Adjust how many lessons you deliver based on student progress throughout the unit, which can be tracked using the formative assessment folder.

Place Value Unit for Years 6-7 – 6C Prime, Composite, Square and Triangular Numbers

Hyperlinked Table of Contents

Place Value Unit for Year 6: Prime, Composite, Square and Triangular Numbers

Curriculum Links for the following lessons

This unit is recommended for Year 6 students (Year 5 for its factors content).

It also includes extension content up to Years 7-8 with its focus on indices, square roots and algebra-related extending prompts.

Australian Curriculum V9 AC9M6N02 and Victorian Curriculum Version 2.0 (VC2M6N02)
Number – Level 6: Identify and describe the properties of prime, composite, square and triangular numbers and use these properties to solve problems and simplify calculations

- using the definition of a prime number to explain why one is not a prime number
- testing numbers by using division to distinguish between prime and composite numbers, recording the results on a number chart to identify any patterns
- representing composite numbers as a product of their factors, including prime factors when necessary and using this form to simplify calculations involving multiplication, such as 15 × 16 as 5 × 3 × 4 × 4, which can be rearranged to simplify calculation to 5 × 4 × 3 × 4 = 20 × 12
- identifying and describing the product of a number with itself as square; for example, 3 × 3 is the same as 3^2
- using spreadsheets to list all the numbers that have up to 3 factors, using combinations of only the first 3 prime numbers, recognising any emerging patterns, making conjectures and experimenting with other combinations

Australian Curriculum V9 AC9M5N02 and Victorian Curriculum Version 2.0 (VC2M5N02)

Number – Level 5: Express natural numbers as products of their factors, recognise multiples and determine if one number is divisible by another

- using a certain number of blocks to form different rectangles and using these to list all possible factors for that number; for example, 12 blocks can form the following rectangles: 1 × 12, 2 × 6 and 3 × 4
- researching divisibility tests and explaining each rule using materials; for example, using base-10 blocks to test if numbers are divisible by 2, 5 and 10
- using divisibility tests to determine if larger numbers are multiples of one-digit numbers; for example, testing if 89 472 is divisible by 3 using 8 + 9 + 4 + 7 + 2 = 30, as 30 is divisible by 3 then 89 472 is a multiple of 3
- demonstrating and reasoning that all multiples can be formed by combining or regrouping; for example, multiples of 7 can be formed by combining a multiple of 2 with the corresponding multiple of 5: 3 × 7 = 3 × 2 + 3 × 5, and 4 × 7 = 4 × 2 + 4 × 5

Australian Curriculum V9 AC9M7N01 and Victorian Curriculum Version 2.0 (VC2M7N01)

Number – Level 7: Describe the relationship between perfect square numbers and square roots, and use squares of numbers and square roots of perfect square numbers to solve problems

- investigating squares of natural numbers from one to 20, and connecting them to visual representations such as dots arranged in a square pattern
- using the square and square root notation, and the distributive property and area diagrams, to calculate the squares of two-digit numbers
- determining between which 2 consecutive natural numbers the square root of a given number lies; for example, 43 is between the square numbers 36 and 49, and therefore between 6 and 7
- generating a list of perfect square numbers and describing any emerging patterns, for example, the last digit of perfect square numbers, or the difference between consecutive square numbers, and recognising the constant second difference
- using the relationship between perfect square numbers and their square roots to determine the perimeter of a square tiled floor using square tiles; for example, an area of floor with 144 square tiles has a perimeter of 48 tile lengths

Australian Curriculum V9 AC9M8N02 and Victorian Curriculum Version 2.0 (VC2M8N02)

Number – Level 8: Establish and apply the exponent laws with positive integer exponents and the zero exponent, using exponent notation with numbers

- recognising the connection between exponent form and expanded form with the exponent laws of product of powers rule, quotient of powers rule, and power of a power rule, for example, $2^3 \times 2^2$ can be represented as $2 \times 2 \times 2 \times 2 \times 2 = 2^5$, and connecting the result to the addition of exponents
- applying the exponent laws of the product of powers rule, quotient of powers rule, power of a power rule and zero exponent individually and in combination; for example, using exponents to determine the effect on the volume of a 2 centimetre cube when the cube is enlarged to a 6 centimetre cube
- using digital tools to systematically explore the application of the exponent laws; observing that the bases need to be the same
- using examples such as $3^4/3^4=1$, and $3^{4-4}=3^0$ to illustrate the necessity that for any non-zero natural number

New WA Curriculum – Number and Algebra – Understanding Number – Year 6: Explore, identify and represent square, prime and composite numbers **in arrays** and explain reasoning.

Extension **– New WA Curriculum – Number and Algebra – Algebraic Techniques – Year 7:** Extend knowledge of factors to represent natural numbers as products of prime factors using index notation as appropriate.

Extension **– New WA Curriculum – Number and Algebra – Algebraic Techniques – Year 7:** Explore and explain connections between square numbers and square roots, cube numbers and cube roots, as products of repeated factors.

NSW Syllabus – Stage 3 – Multiplicative relations A

Determine products and factors

- Use the term *product* to describe the result of multiplying 2 or more numbers
- Model different ways to show a whole number as a product (Reasons about structure)
- Determine factors for a given whole number
- Determine whether a number is prime, composite or neither (0 or 1)

NSW Syllabus – Stage 4 – Indices

Apply index notation to represent whole numbers as products of powers of prime numbers

- Describe numbers written in index form using terms such as *base*, *power*, *index* and *exponent*
- Represent numbers in index notation limited to positive powers
- Represent in expanded form and evaluate numbers expressed in index notation, including powers of 10
- Apply the order of operations to evaluate expressions involving indices
- Determine and apply tests for divisibility for 2, 3, 4, 5, 6 and 10
- Represent a whole number greater than one as a product of its prime factors, using index notation where appropriate

Examine cube roots and square roots

- Use the notations for square root and cube root
- Recognise and describe the relationship between squares and square roots, and cubes and cube roots for positive numbers
- Estimate the square root of any non-square whole number and the cube root of any non-cube whole number, then check using a calculator
- Identify and describe exact and approximate solutions in the context of square roots and cube roots
- Apply the order of operations to evaluate expressions involving square roots, cube roots, square numbers and cube numbers

Use index notation to establish the index laws with positive-integer indices and the zero index

- Establish the multiplication, division and the power of a power index laws, by expressing each number in expanded form with numerical bases and positive-integer indices
- Verify through numerical examples that
- Establish the meaning of the zero index
- Apply index laws to simplify and evaluate expressions with numerical bases

Formative Assessment

A formative assessment cross-check is available in this unit's folder with progressive learning goals and specific success criteria for this unit.

Formative Assessment – Ongoing Cross-Check for Prime, Composite, Square and Triangular for Years 6-7						
Year 6-7 Place Value **Students**	Can explain the difference between prime and composite numbers **L6**	Can show a number as a product of its factors **L6**	Can identify square numbers and justify why they are classified as such **L6**	Can identify triangular numbers and justify why they are classified as such **L6**	Can show a number as a product of its prime factors (prime factorisation), including accurate use of index notation **L7**	Uses prime factors to calculate GCF and LCM **L7**

Prime Composite Square Triangular Year 6C Lesson 1	**The Ice-Cream Store** **Learning intention: Link the concept of prime and composite numbers to arrays and times tables. Use this to determine which numbers are prime and which are composite, explaining why.** **Maths vocabulary: prime number, composite number, factors, array, times tables, divisible, remainders**
Literacy Link: *Bleezer's Ice Cream Poem* by Jack Prelutsky, 1940, YouTube of the poem: https://www.youtube.com/watch?v=YuPlUQvViX8. Buddy read the poem for fluency. As a writing session prior to maths, students create their own store and poem, brainstorming whacky flavours with alliterations like 'Cotton Candy Carrot Custard' using the same list poem format.	**Lesson summary: Students create an ice-cream store and, in the process, investigate which numbers can be neatly arranged into lots of different arrays or combinations (composite) and which numbers only make one long row or one tall column (prime).** **Materials:** • Counters – square if possible, or circular. • 120 chart. • *List of prime numbers up to 120 for teacher reference:* 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113. **Best set-up:** Model how to use counters to investigate a few different prime and composite numbers (7, 12, 13) as a class. Then students continue the investigation, marking composite numbers in green on the 120 chart, and prime numbers in red. Finally, aim to find a composite number with lots of factors that would be a reasonable choice for an amount of ice-cream flavours served daily at their new shop, and prove why they chose it. **How are ice-cream stores usually set up? As an array!** 

Literacy Link*:* Page 16 of the *26 Storey Treehouse* by Andy Griffiths with 78 crazy flavours served by a robot (YouTube link: https://www.youtube.com/watch?v=85lXNTcFLIM).

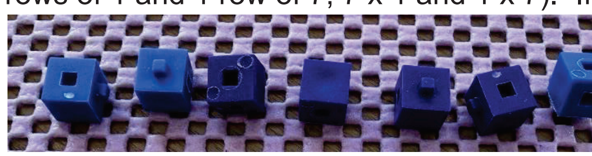
Continuation of the hook for maximum engagement: Students are given the creative freedom to set up their own ice-cream store. Within the hook time, brainstorm a name for their shop and a quick list of their favourite or craziest flavours, particularly after watching the *Ebenezer Bleezer* video clip from the hook, or reading the poem about whacky ice-cream flavours.

Problem-solving challenge and investigation for students: As a new ice-cream store owner, today your job is to solve these two big questions:

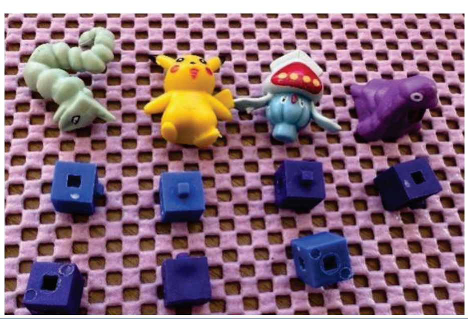
1. If you chose to have 17, 19 or 23 flavours, how many options would you have in terms of setting up your store as an array?
2. What do you think is the best number of flavours for your store? Consider that you want to:
   - Rearrange your array every day to keep customers interested (in retail this is called 'fronting up' so that customers want to buy heaps of your products), AND
   - Offer a variety of options for customers (but not too many so you need a robot to serve them all like in the *26 Storey Treehouse* (see hook left).

Modelling: Model the difference between a prime and composite number using counters to represent tubs of ice-cream and by linking to the concept of arrays.

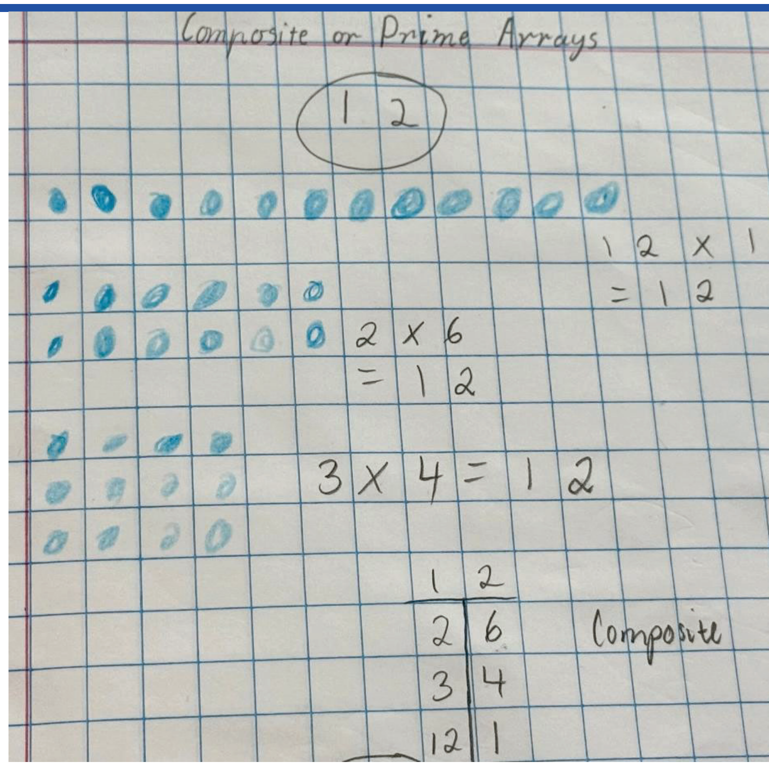
7 is a **prime** number. We can tell because it can only be arranged into one long row or one tall column. No matter how hard I try, I cannot arrange it into a rectangle or a square, so it only makes two **arrays**, and only has two **factors** (itself and 1, since it is 7 rows of 1 and 1 row of 7, 7 x 1 and 1 x 7). If you find a number is **composite**, it has many combinations for different arrays, not just 1 row or 1 column.



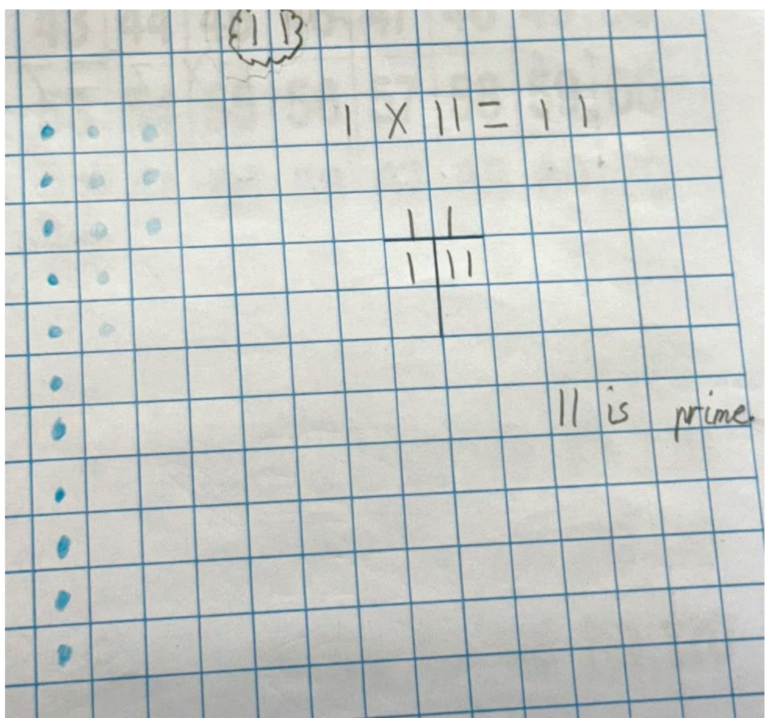
Link to **divisibility** as well, by placing characters alongside the array, as if they were sharing the ice-creams. 7 cannot be shared equally between any number of people, apart from 1 person or 7 people. It is not divisible by 2, or 3, or 4, or 5, or 6, and it just keeps creating **remainders**!



Literacy Link: Read the numeracy book *Bean 13*, then try to make 13 into arrays and share it, just like in the story. Another prime number – it just does not work!



Initial practice – investigating 12 and 13 with counters to form arrays

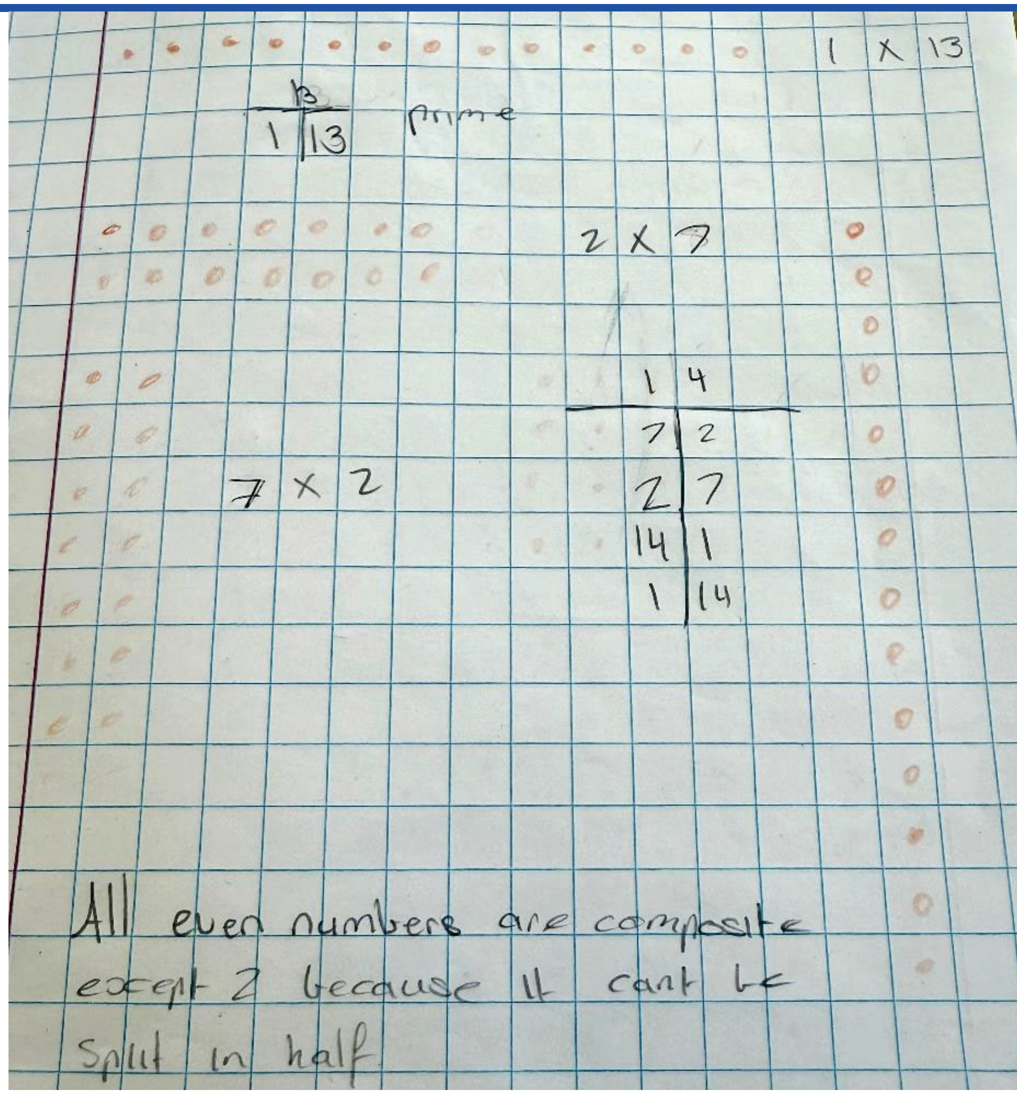


Investigating the factors of 12 using donuts, from the staffroom morning tea leftovers, as an extra engaging manipulative:

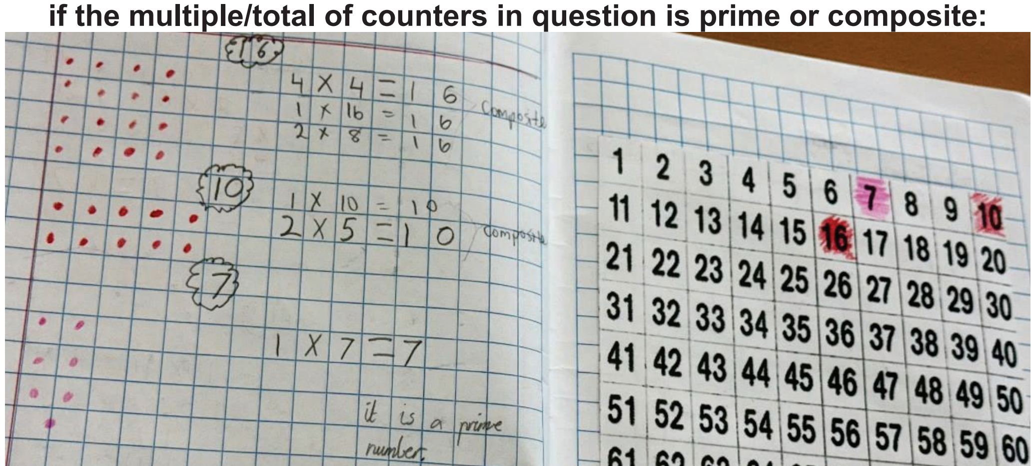
3 rows of 4 is 12, 3 x 4 = 12, so 3 and 4 are factors of 12

2 rows of 6 is 12, 2 x 6 = 12, so 2 and 6 are factors of 12

1 row of 12 is 12, 1 x 12 = 12, so 1 and 12 are factors of 1 2



Introductory counters tests, with students making arrays to determine if the multiple/total of counters in question is prime or composite:

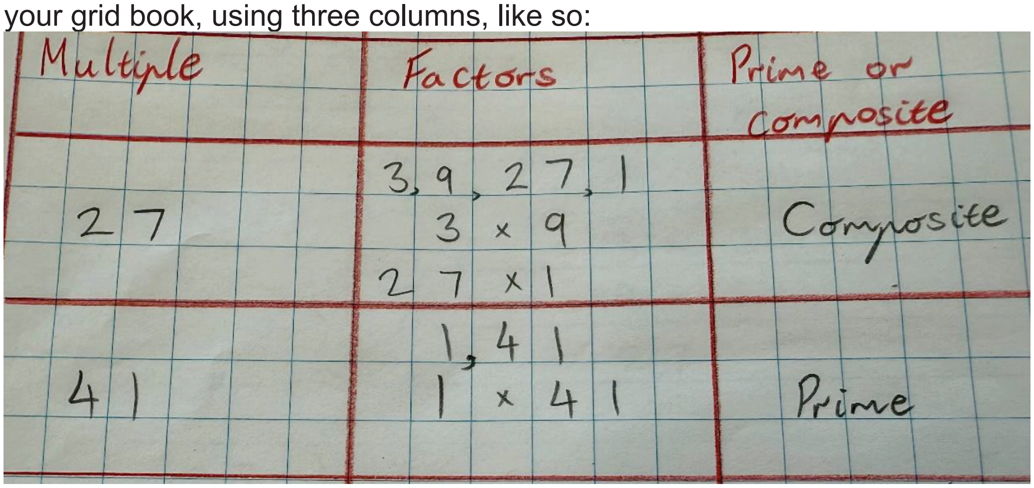


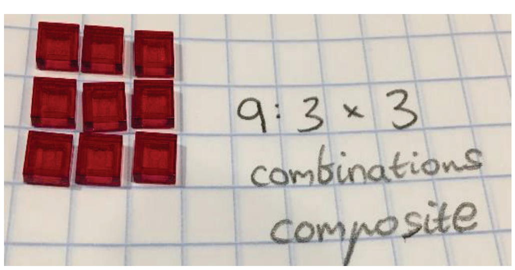
Recording:

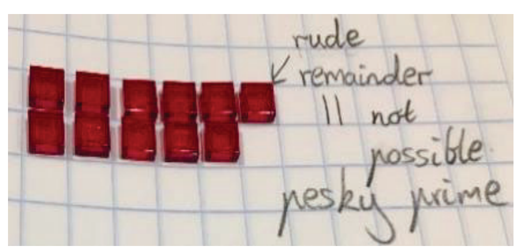
As you trial different numbers, mark them in **red** on the 120 chart if they are **prime**. That is, if you can only make one long row or one tall column.

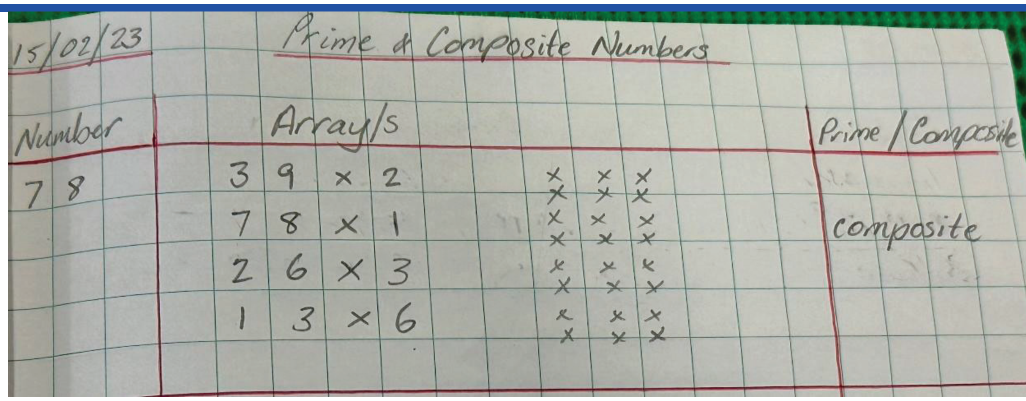
Shade **composite numbers in green** on the chart.

Also record all the combinations and factors you discover for each multiple in your grid book, using three columns, like so:

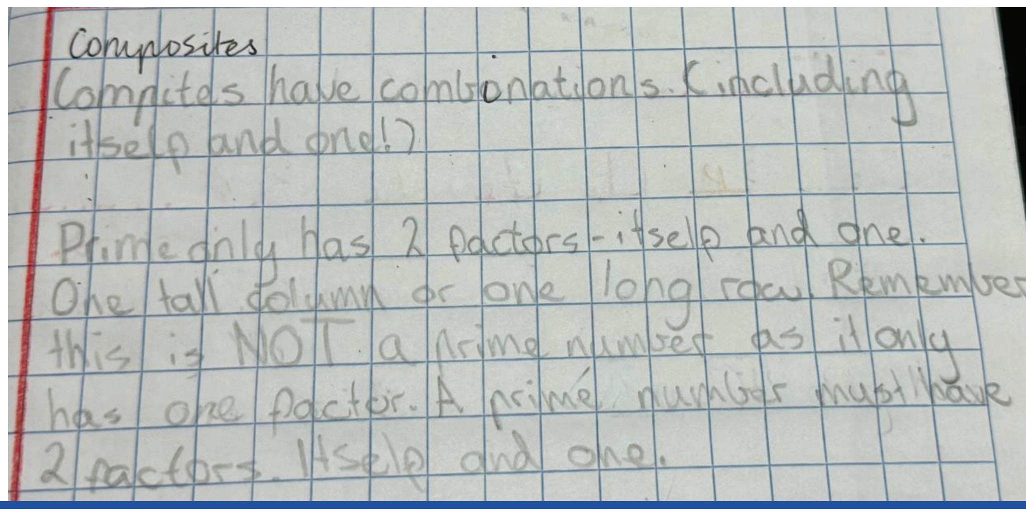


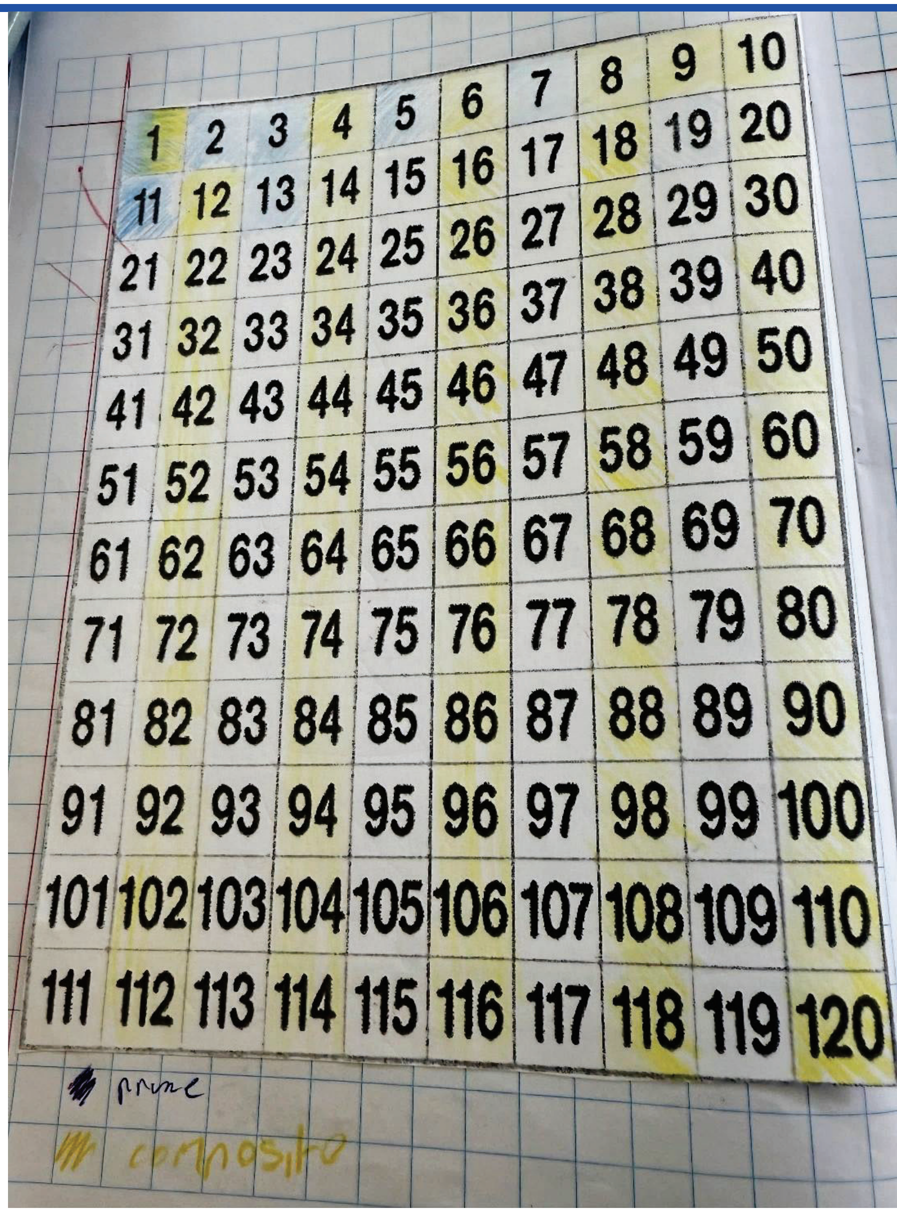




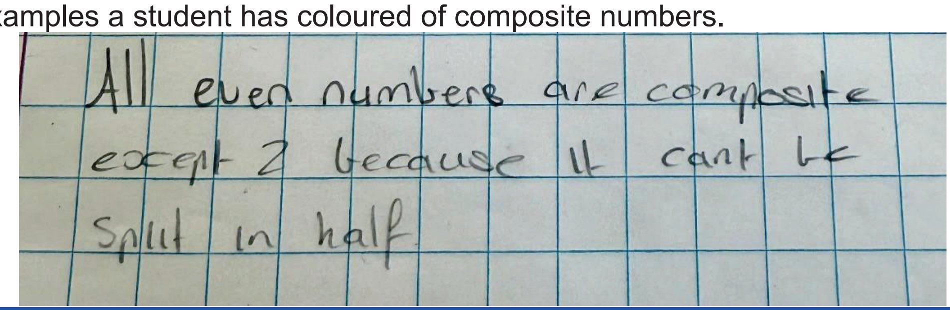


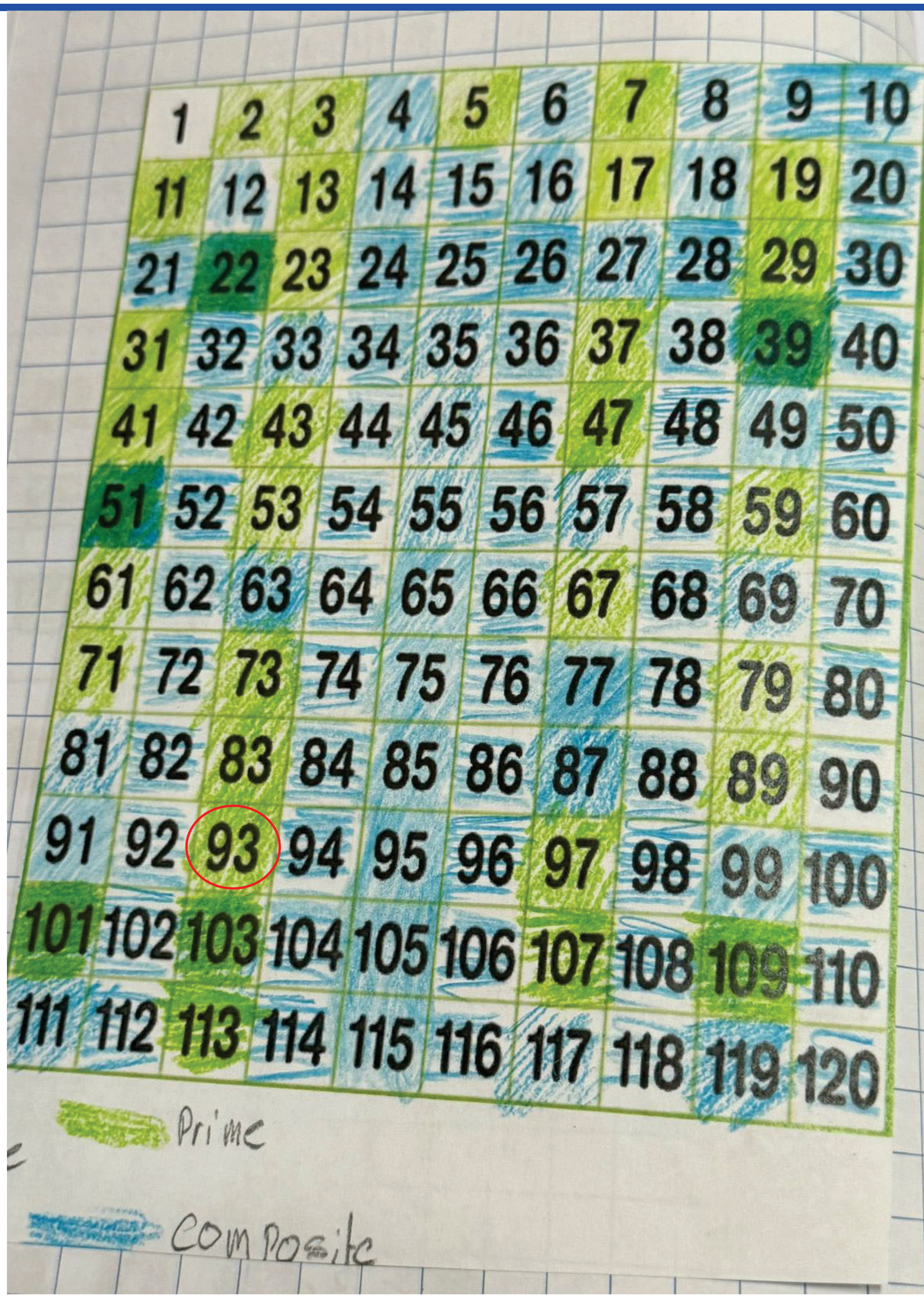
Student recording and recorded definition





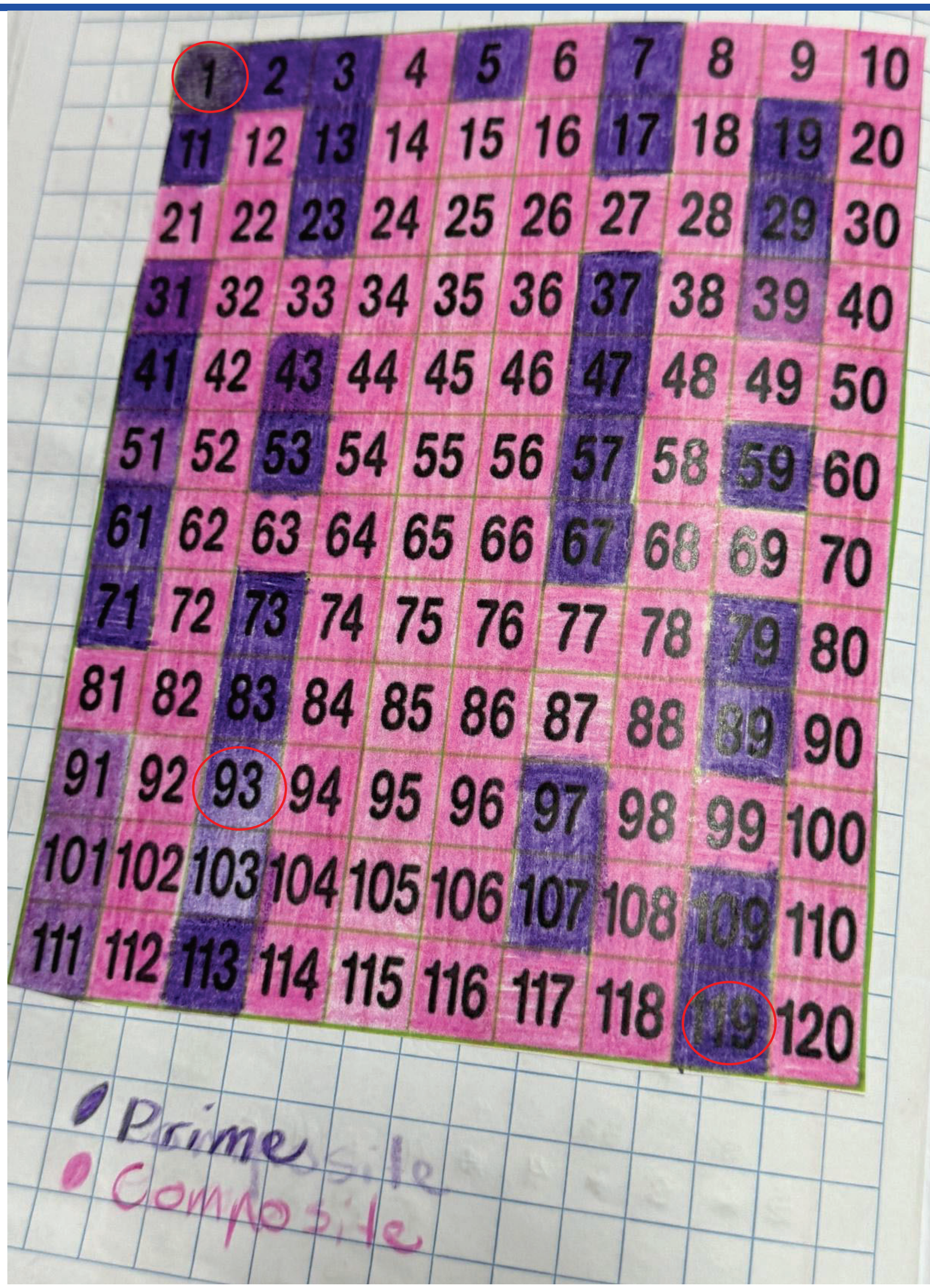
Towards the start of the task, students focused on ‘knocking out’ as many composites as possible by finding patterns. This included divisibility by 2 or 5, as shown above with the ‘2’ pattern highlighted in yellow as the first examples a student has coloured of composite numbers.





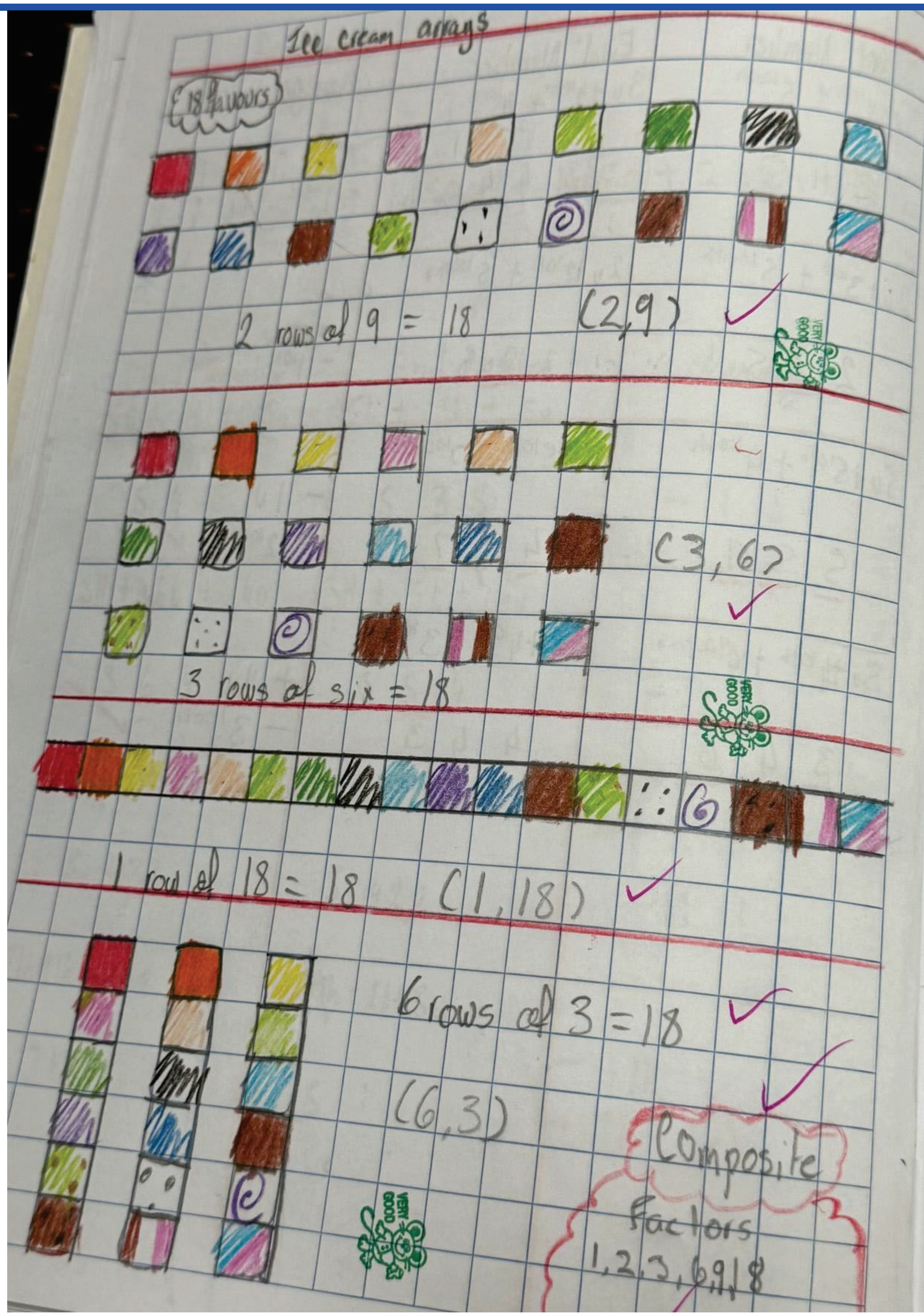
Prime/composite 120 chart student work sample of recording, where most are correct, although students tend to be tricked by multiples of 3, such as 39 and 51, and this student also changed their mind about 22 as well **(dark green grids show where the student has changed their mind).**

Love that '1' has been left white – neither prime, nor composite.



Another example where the student has changed their mind and tried to change the colour of 39 and 51 in particular.

The student has also changed their mind about ‘1,’ attempting to change its colour to be neither prime, nor composite.



After investigating many potential totals for their ice-cream store, students finish the lesson by choosing a total for their ice-cream store, and justifying why they chose it.

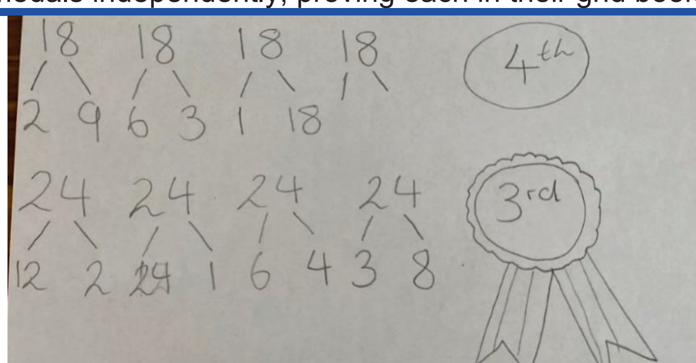
	Questioning and patterns to notice: • What do you notice about the composite numbers? - Many are even numbers, but not all composites are even (some can be odd). - All are in a times table. - All are divisible by at least one other number in addition to 1 and itself. - The higher the numbers become, the less prime numbers you find, because there are often more factors or ways to arrange that total into arrays. For example, 2, 3, 5, 7, 11 and 13 are prime numbers. That is a lot of prime numbers very early on. However, as you go up the 120 chart (and beyond), you find less and less! • How can you work out which numbers will be factors of that total? You can look at the rows and columns of the arrays, but what else can you think about? Do factors relate to the times tables? • Is there a quicker way to work out whether a number is prime or composite (without using the counters)? Use the times tables! If a number is in the 2 times tables (except for '2' itself), it is composite! If it is in the 3 times tables (except for '3' itself), it will be composite, and so on. If you know the times tables strategy, you can check. For example, if the number can be halved, it will be in the 2 times tables. If it can be halved and halved again (without creating a remainder or decimal), it will be in the 4 times table, and so on by applying the strategies for each times table and divisibility pattern. For example, if the sum of the digits in the number adds to 9, then it will have 9 as a factor, so it will definitely be composite (it will also have 3 as a factor, since 3 is a factor of 9). **Reflection problem-solving questions:** • What did you notice about all the composite numbers? What do all of these have in common? Every composite number is part of a times table (except for the first number in that times table, like 7). • Can a prime number be even? Why or why not? Are all composite numbers even? Prime numbers are not even (except the number 2), because then they could make arrays with two rows, which would be an extra array combination (so not prime). Composite numbers can be even or odd (15 is odd, but it is still composite, because 3 rows of 5 makes an array that is not just 1 x 15 or 15 x 1).

	Misconception alert: Some students think that composite numbers must be even, because so many of them are and because all two-digit prime numbers are odd, but this is not the case. • If a number is divisible by 2, will it be prime or composite? How can you be sure? Because if it is divisible by 2, it can be arranged into an array of 2 rows – 1 row for each person – so it must be composite. This applies for every number except '2' itself, which is divisible by 2 but is prime. • What about 1? Is it prime or composite? What about 0? 1 is not considered prime, as a prime number has 2 factors, and it has only 1 factor. For further discussion, read: https://blogs.scientificamerican.com/roots-of-unity/why-isnt-1-a-prime-number/. 0 is also not prime, but nor is it composite, so both are considered non-prime.
	Support: Only provide a limited number of counters, for example, investigate numbers up to 40, rather than up to 120. For numbers above 40, the times tables is a more efficient strategy than counters).
	Mid-range students and extension 1: Do not require students to use counters after they can explain and justify some prime and composite numbers using arrays if they can instead use their times tables and have made this connection. Counters in arrays are critical to establish the initial definitions and justifications for prime and composite numbers, and factors as well, but the use of these slows down the investigation process if times tables are an alternate strategy.
	Extension – Largest Prime Challenge: First justify their choice of their total number of ice-cream flavours and use prime/composite reasoning to explain this choice. Next, challenge these students to find the largest prime number that exists. For example, 359 is prime, can you find a larger one? Aim to find a higher prime number than the extension student sitting beside them who is attempting the same challenge (pair your high extension students together to challenge one another – like-ability partners on mixed-ability tables). This will be challenging, because it is hard to formulate a specific strategy to work this out. Even so, challenge students to try to come up with a strategy. How can you be sure that the number is prime? What tests does it need to pass (for example, the number must fail all divisibility tests, except for divisibility by 1). It must also be odd (anything ending in 2, 4, 6, 8 and 0 is

	eliminated from consideration). Any number ending in 5 and 0 is also eliminated, because it is divisible by 5. Likewise, any numbers whose digits sum to 3 or 9 are eliminated (divisibility by 3 and 9). Therefore, one strategy could be to focus on numbers that end in 3, 6, 7, 8 and 9. After some time to try the challenge themselves, read through these tips: https://www.f1gmat.com/how-to-identify-a-prime-number The Sieve of Eratosthenes is the highly regarded strategy for finding primes under 10 billion: https://primes.utm.edu/prove/prove2_1.html Also read about the history of mathematicians' quest for the highest prime number that exists: https://slate.com/technology/2018/01/the-worlds-largest-prime-number-has-23249425-digits-heres-why-you-should-care.html
	Extension – Follow-on from *Extension 2* – Largest Twin Primes: Find the largest twin primes – two prime numbers that are only two apart (for example, 41 and 43). Aim to find a larger set than your extension buddy finds. **Questioning prompt:** Why can't the numbers be one apart? (Then one would be even!)

Shade the composite numbers in green and the prime numbers in red. Keep investigating composite and prime numbers, while you trial the best number of ice-cream flavours.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

Prime Composite Square Triangular Year 6C Lesson 2	**The Composite Number Olympics** **Learning intention: Use a range of different recording methods to brainstorm factors for a multiple, aiming to find numbers with lots of factors within a set range (0 to 120).** **Maths vocabulary: highly composite number, factor tree, factors, product, multiple, prime, composite, double it halve it strategy**
YouTube hook – Australia's top 10 Olympic moments: https://www.facebook.com/olympics/videos/10-great-australian-olympic-moments/2720076434762798/ **Acts of great human spirit at the Olympics:** https://www.youtube.com/watch?v=Gh6b0b0Lj84 **7 interesting facts about Olympic medals:** https://www.visitcos.com/blog/seven-things-you-didnt-know-about-olympic-medals/	**Lesson summary: Students search for highly composite numbers in the form of 'gold medal' or 'golden' composite numbers, often earning silver or bronze medals as a consolation prize when they have not located a gold. Students can use a range of recording methods (factor trees, T-charts, factor fireworks and rainbows) to brainstorm the factors for a number, ultimately aiming to identify multiples up to 120 that have 12 or more factors.** **Materials:** • **Composite medals scoring sheet** for students to record each medal contender, after proving it in their grid book. • A **summarised teacher answer sheet** is also available for rapid immediate feedback to award medals on-the-spot throughout the session. Even more comprehensive answer sheet for teachers to check a student has discovered all factors of a given number: https://pittsburgusd.net/documents/Employee-Essentials/Resources-for-Educators/Elementary-Teacher-Resources/Math-Files-K-5th-Grade/Factor-Chart_noprimes.pdf (this sheet does not list primes, as the only factors for those are 1 and the number itself). • Yellow counters as gold medals, clear or blue as silver, and orange as bronze, or similar. Alternatively, students can simply draw a decorative medal in their books beside their working out when they have earned each medal and checked it with the umpire (teacher), as shown below. There are alternative printable medal templates if you wish – students can write the number that scored the medal inside it. **Best set-up:** Find one or two 'medals' as a class, showing students how to prove the number of factors using a T-chart, factor firework, factor rainbow or factor tree – whichever recording method they prefer. Challenge students to earn more medals independently, proving each in their grid book. 

Key instructions

Earning medals: Explain to students that they have entered the composite medal Olympics! The numbers 0 to 120 are in play.

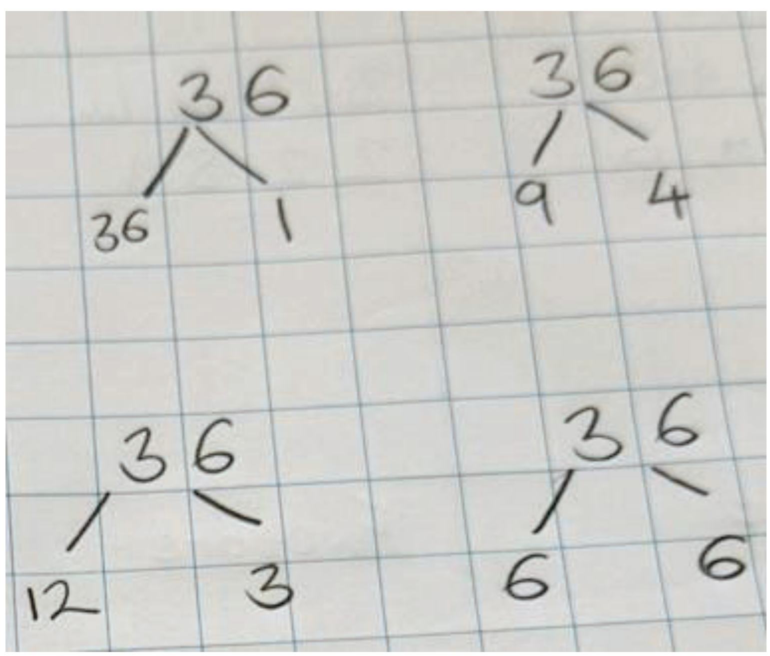
The goal is to search for **highly composite numbers** – numbers up to 120 that have 12 factors or more, to earn a gold medal! Essentially, these are 'golden' composite numbers, or highly composite numbers.

Factor x **factor** = **product/multiple**, so students are looking for products/multiples that have lots of different factors.

Students choose a number they believe may be a medal contender because it has lots of factors (perhaps they believe it is in lots of times tables, or highly divisible), then test it using factor trees, or T-chart brainstorms, or factor fireworks, or factor rainbows, or similar (all shown on the pages that follow).

- If the multiple has 12 factors or more, submit it to the umpire (hands-on heads or list it on the score sheet for checking), to collect a gold medal!
- If it has 10 or 11 factors, silver!
- 8 or 9 factors is bronze!
- 6 or more is a ribbon.

Students can draw their own medals on the page, or use the printable medal templates, or be given counters in colours matching each medal type.

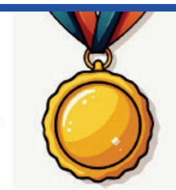


Factor tree recording method

Name:

Composite Olympics

Only the numbers 0-120 are eligible to complete!



Points	Medal awarded	Numbers that qualify (must prove working out in your grid book)
5 points	Gold medals ***12 factors or more***	
4 points	Silver medals ***10 or 11 factors***	
3 points	Bronze medals ***8 or 9 factors***	
2 points	Fourth ribbons ***6 or 7 factors***	
1 point	Prime trophies ***Prime numbers***	

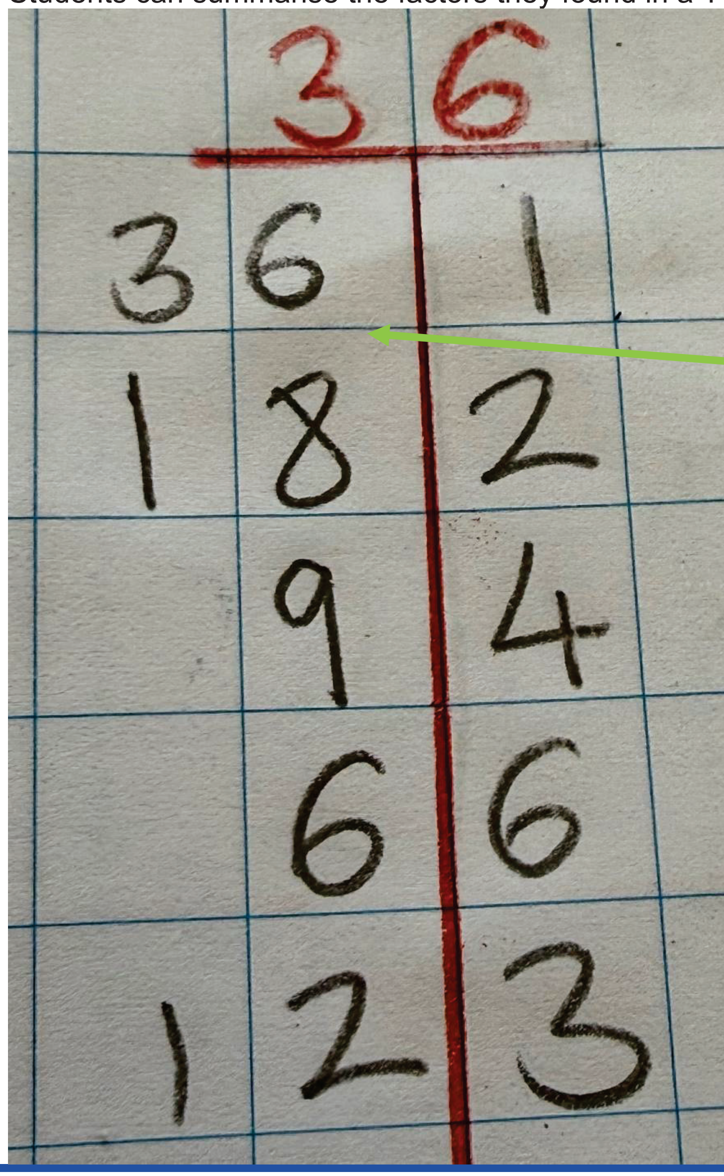
Recording template and teacher answer sheet available

Answer Sheet

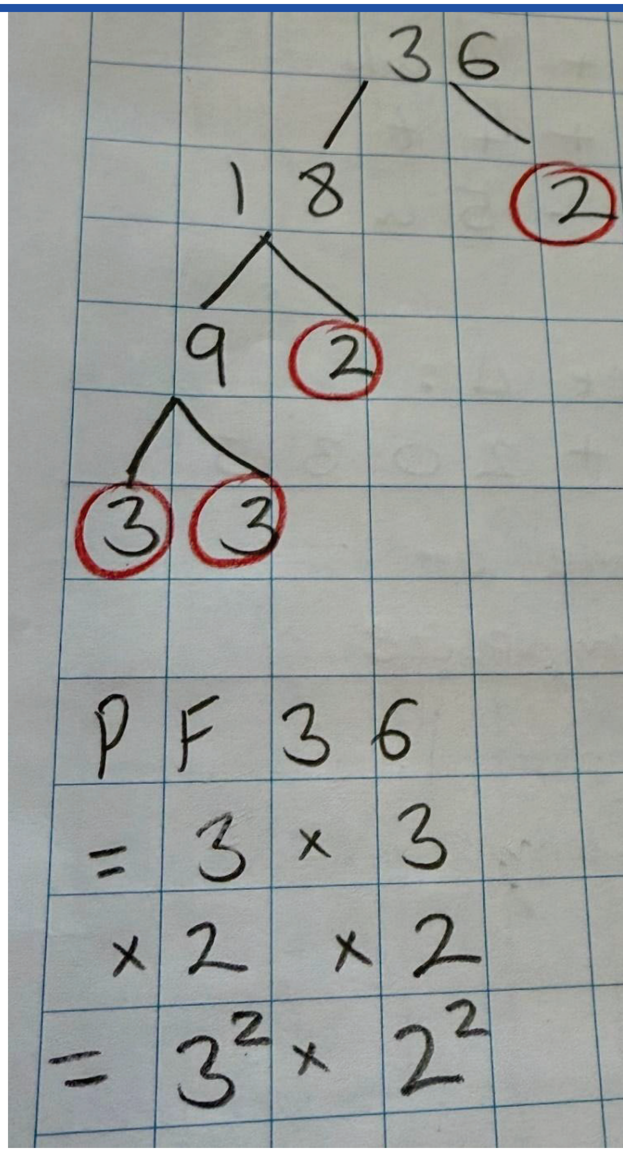
Points	Medal awarded	Numbers that qualify
5 points	Gold medal *12 factors or more*	60, 72, 84, 90, 96, 108, 120
4 points	Silver medal *10 or 11 factors*	48, 80, 112
3 points	Bronze medal *8 or 9 factors*	24, 30, 36, 40, 42, 54, 56, 66, 70, 78, 88, 100, 102, 104, 105, 110, 114
2 points	Fourth ribbon	12, 18, 20, 28, 32, 44,

Critical tip: One strategy to find more factors for a multiple is to **halve one factor, and double the other**, just like the multiplication strategy.

Students can summarise the factors they found in a T-chart:



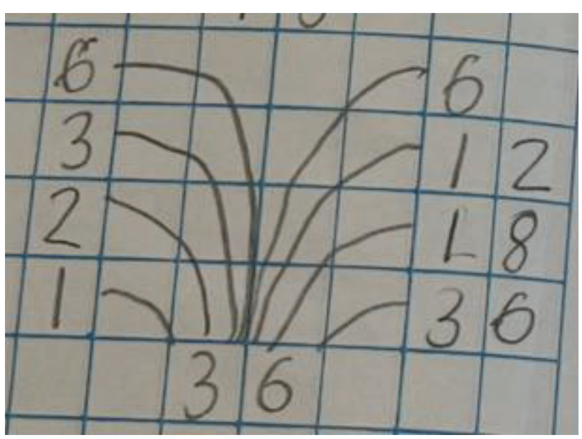
Halve one factor, double the other and it will produce a new pair of factors.



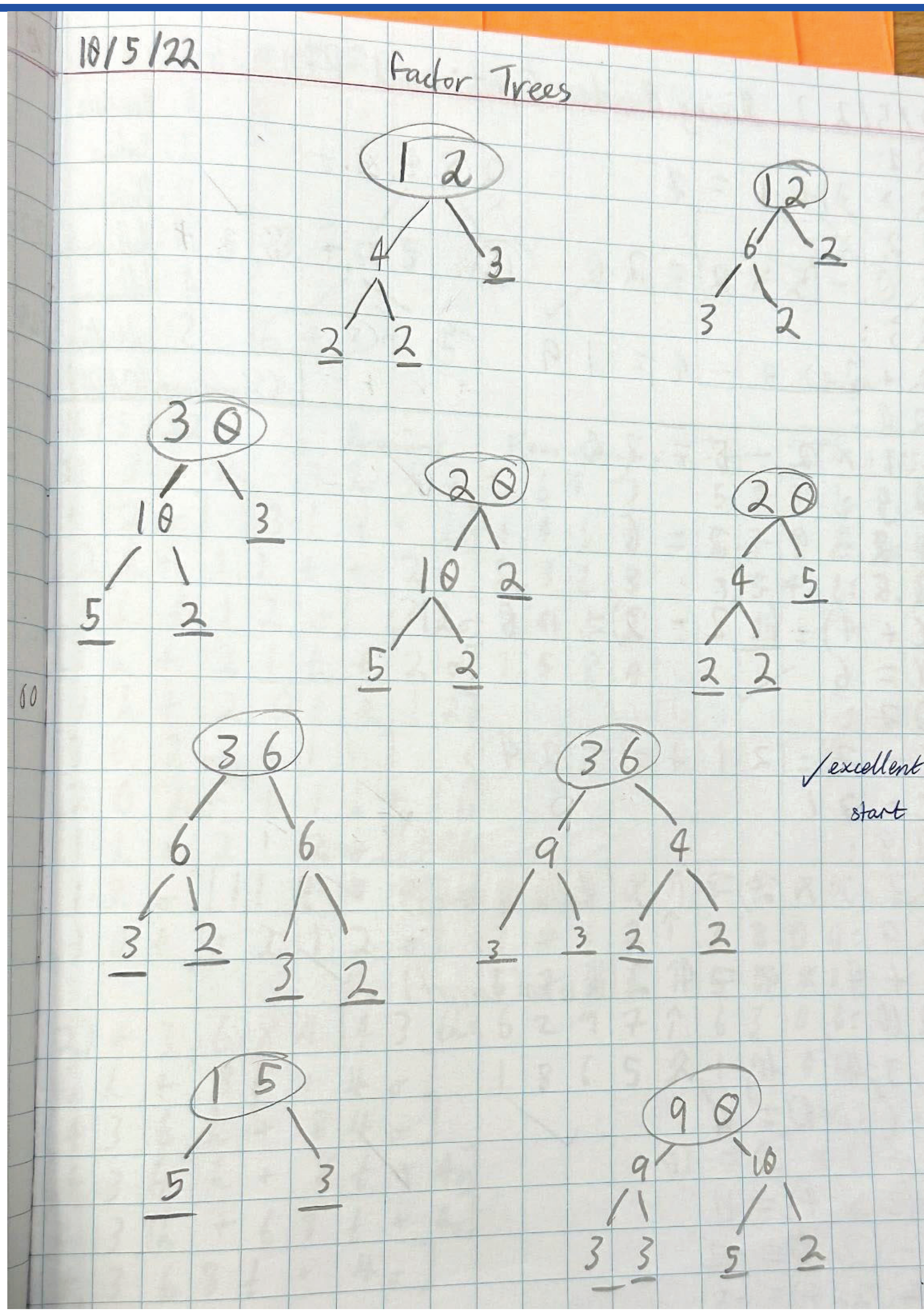
Full factor tree leading to the prime factorisation of the multiple being tested.

Fair play warning: Pre-warn students not to share/cheat by whispering successfully numbers to one another – don't help your competitors beat you – you are at the Olympics, so they must earn their own medals!

If you are caught cheating, all your medals will be confiscated, similar to the countries that are caught taking performance-enhancing drugs.



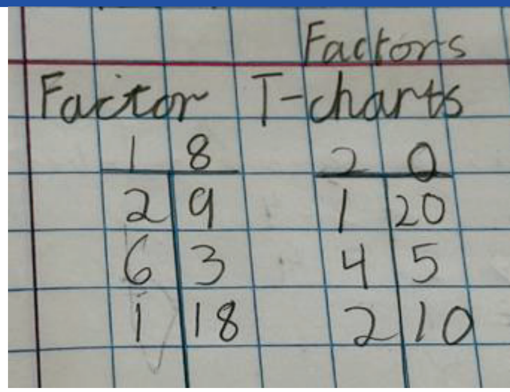
Factor fireworks for 36



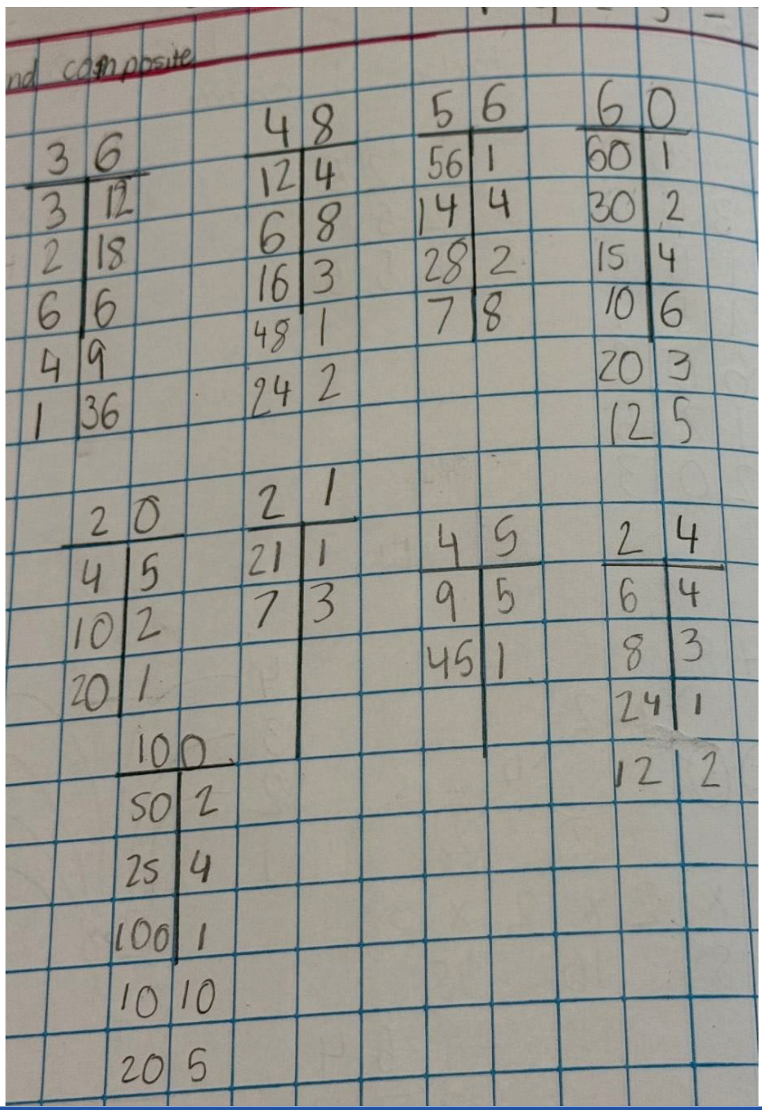
Questioning prompts:

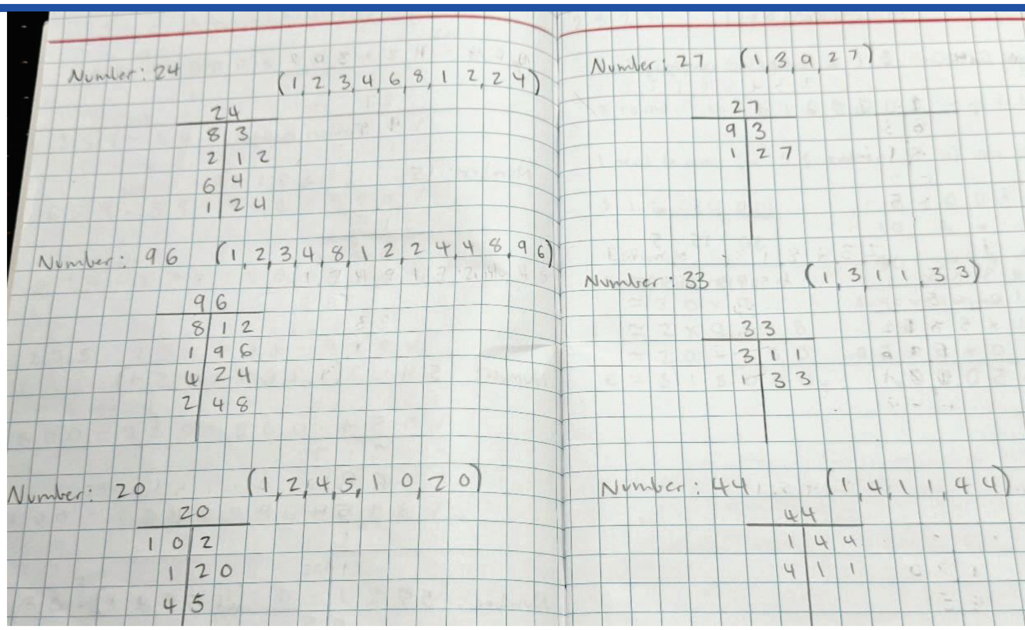
- How did you choose which numbers to try? What sort of strategies did you use, or what did you think about? (Even/odd, divisibility, multiplicative families, skip-counting patterns).
- Why do some numbers have an odd number of factors? That does not make sense, because factors must come in pairs (the rows and columns of the array). How can this be possible?

Alternative recording option – Factor T-Charts

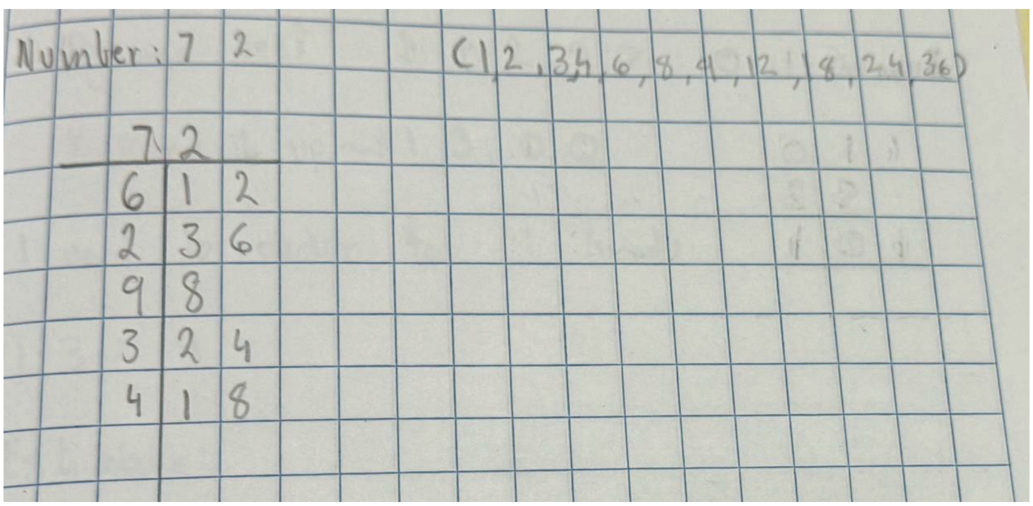


Using T-charts to brainstorm factors, particularly the double one halve the other, or triple one third the other (2 x 9, make it 6 x 3 by tripling the 2 and finding one third of the 9).

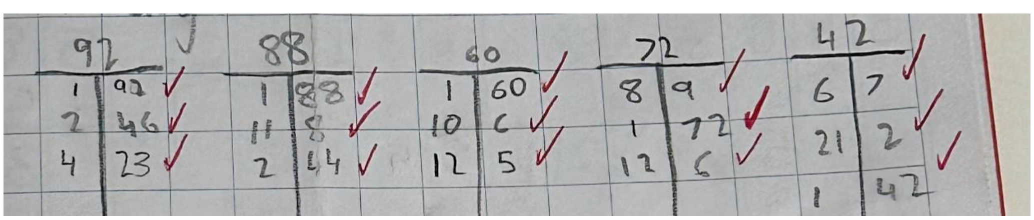




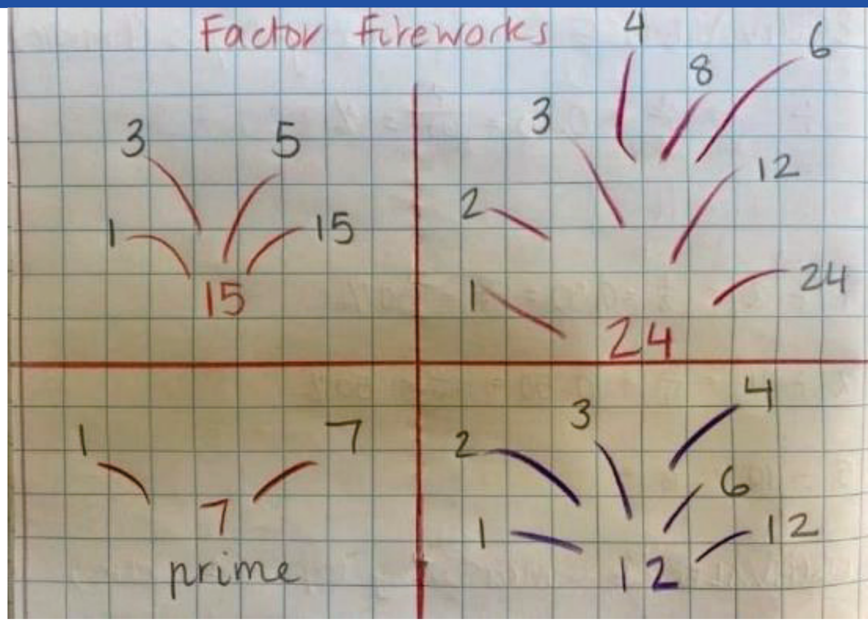
Student work samples



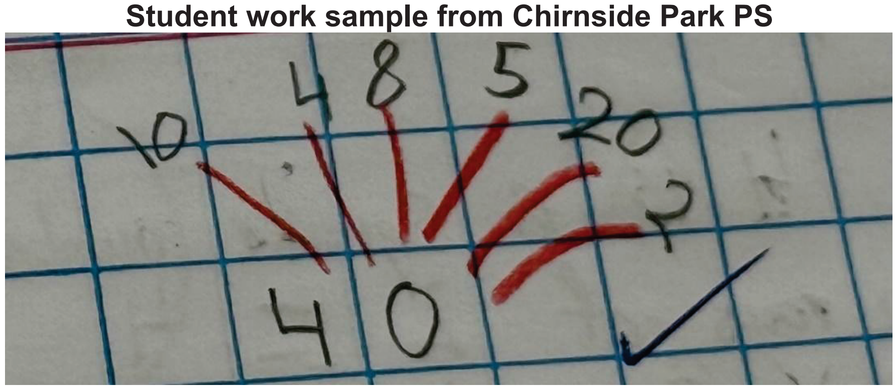
Student work samples

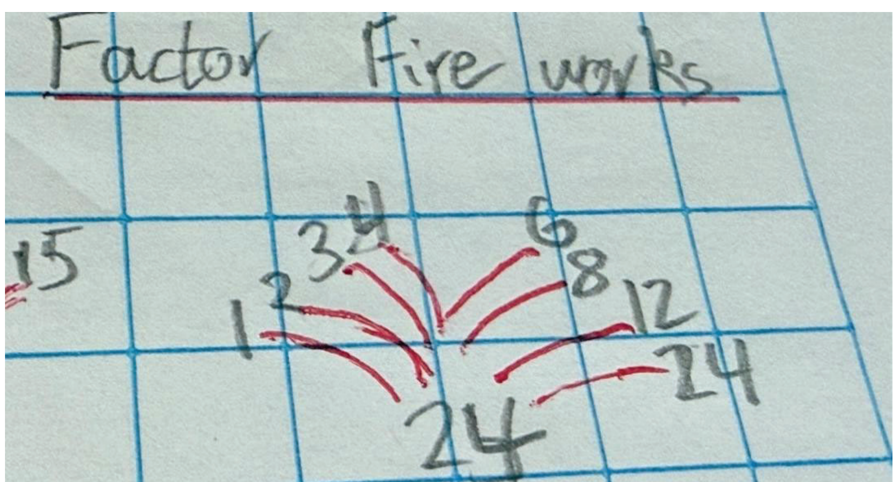


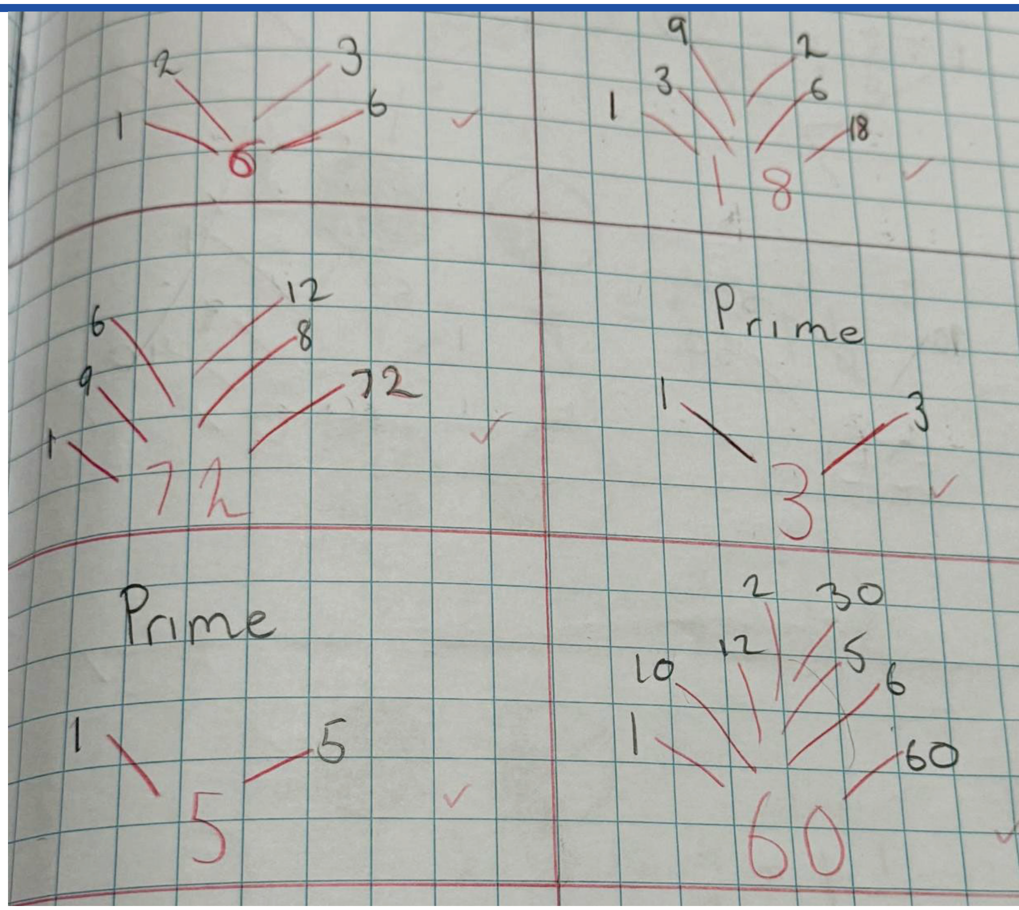
YouTube hook – fireworks: The best fireworks I have ever seen in my life were at Amalfi: https://www.youtube.com/watch?v=LikbtDjn6nM The fireworks at Amalfi on their patron saint day are so intense and so close to the audience that it almost feels unsafe!	**Alternative recording option – Factor Fireworks**



Student work sample from Chirnside Park PS

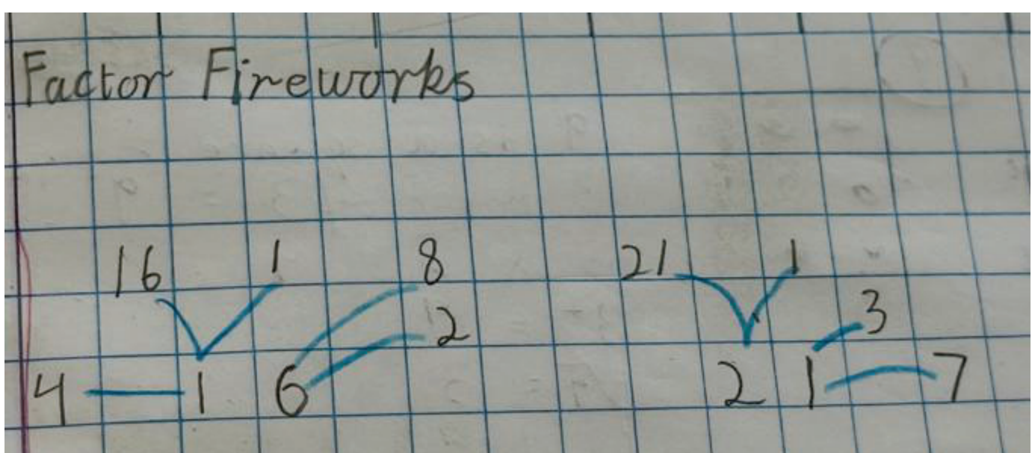


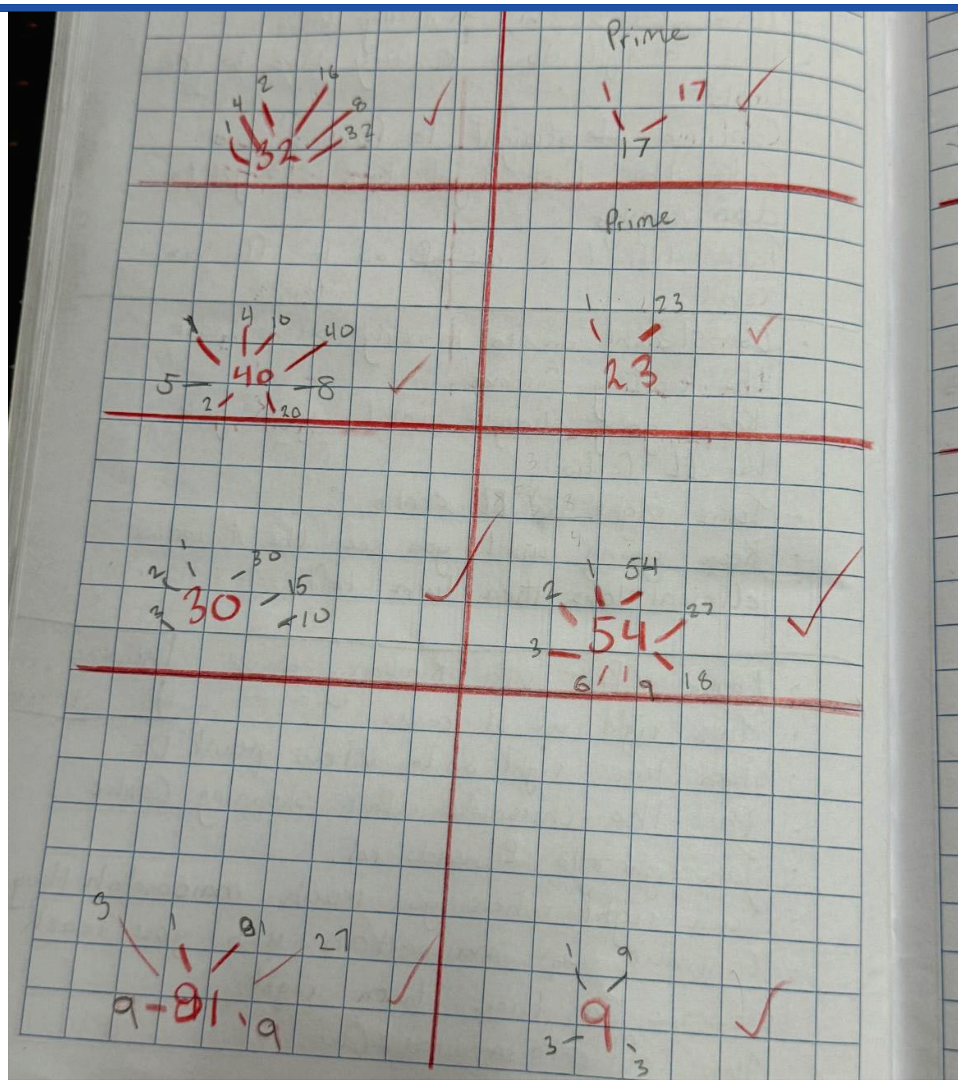




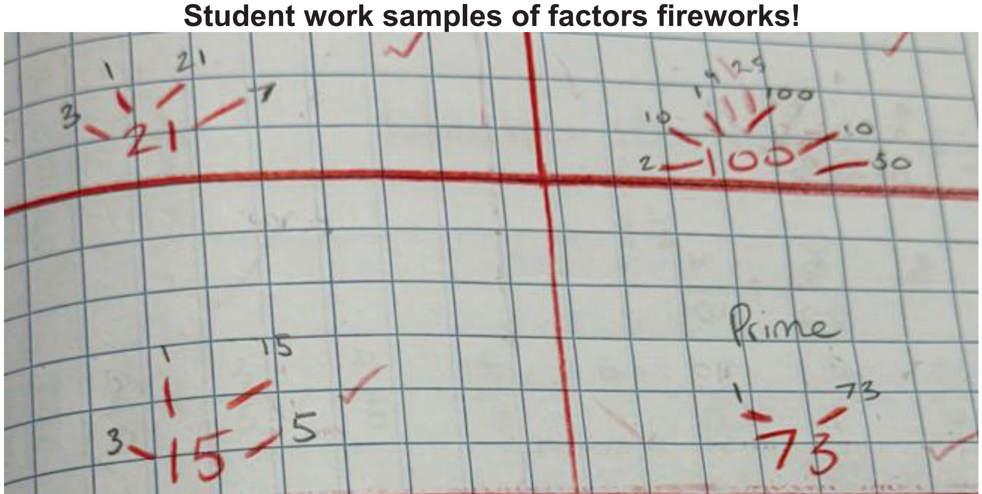
Which numbers produce the most spectacular fireworks?
Those are the golden/highly composite numbers!

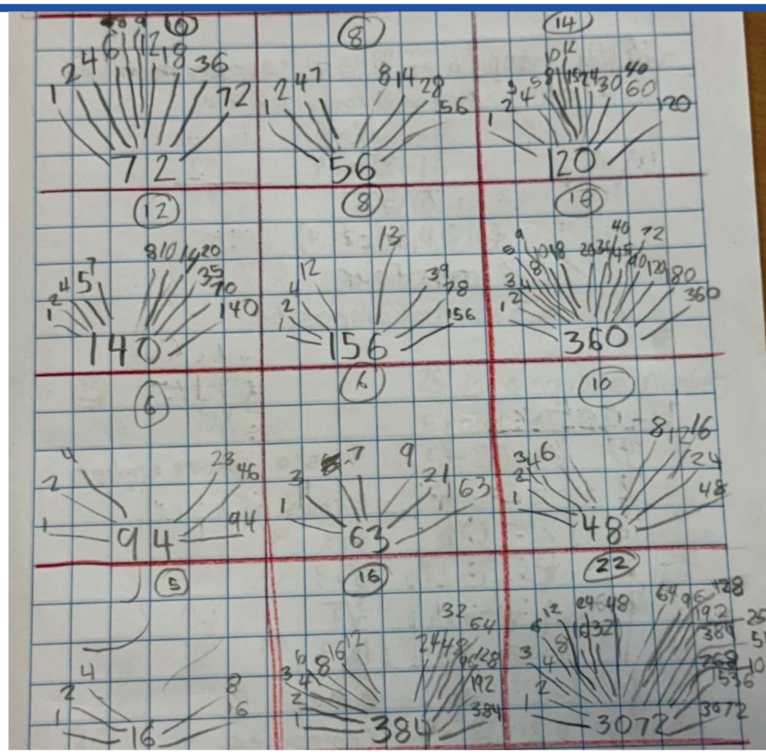
Which make terribly disappointing fireworks?
Those are prime!





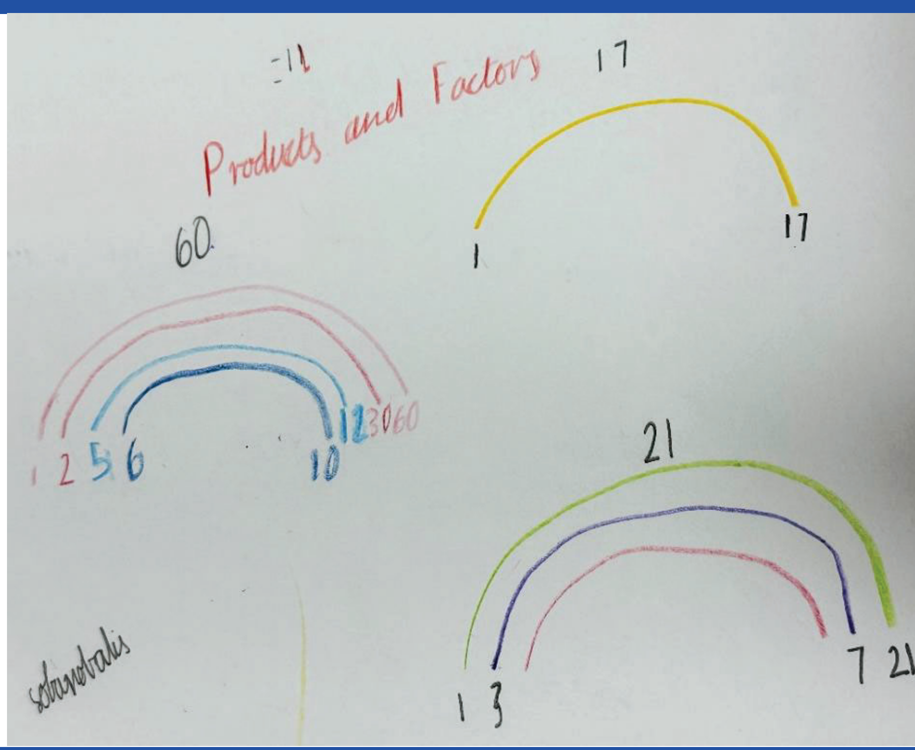
Student work samples of factors fireworks!



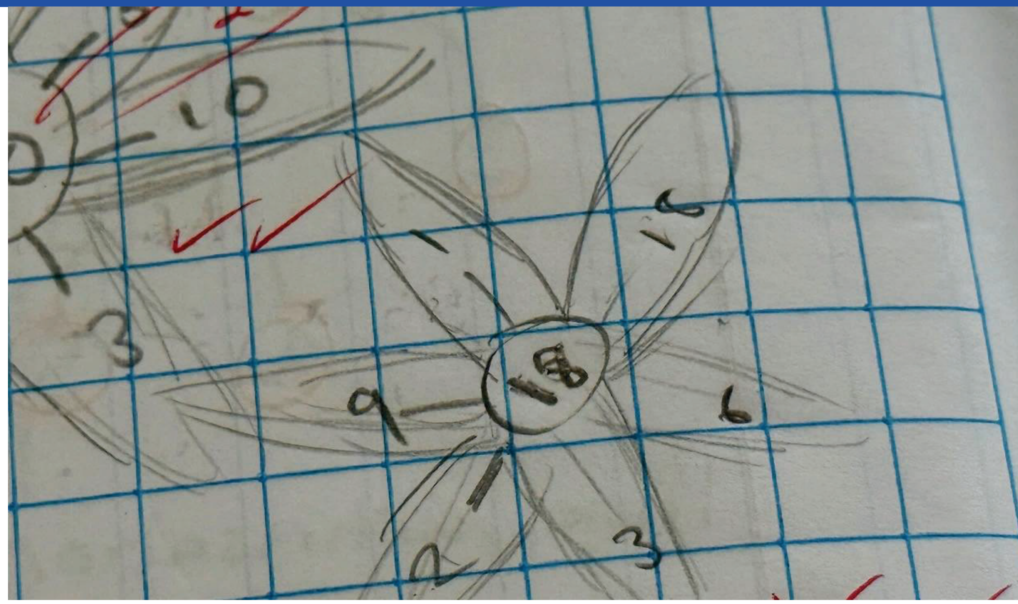


Student attempting larger multiples (higher range) extension

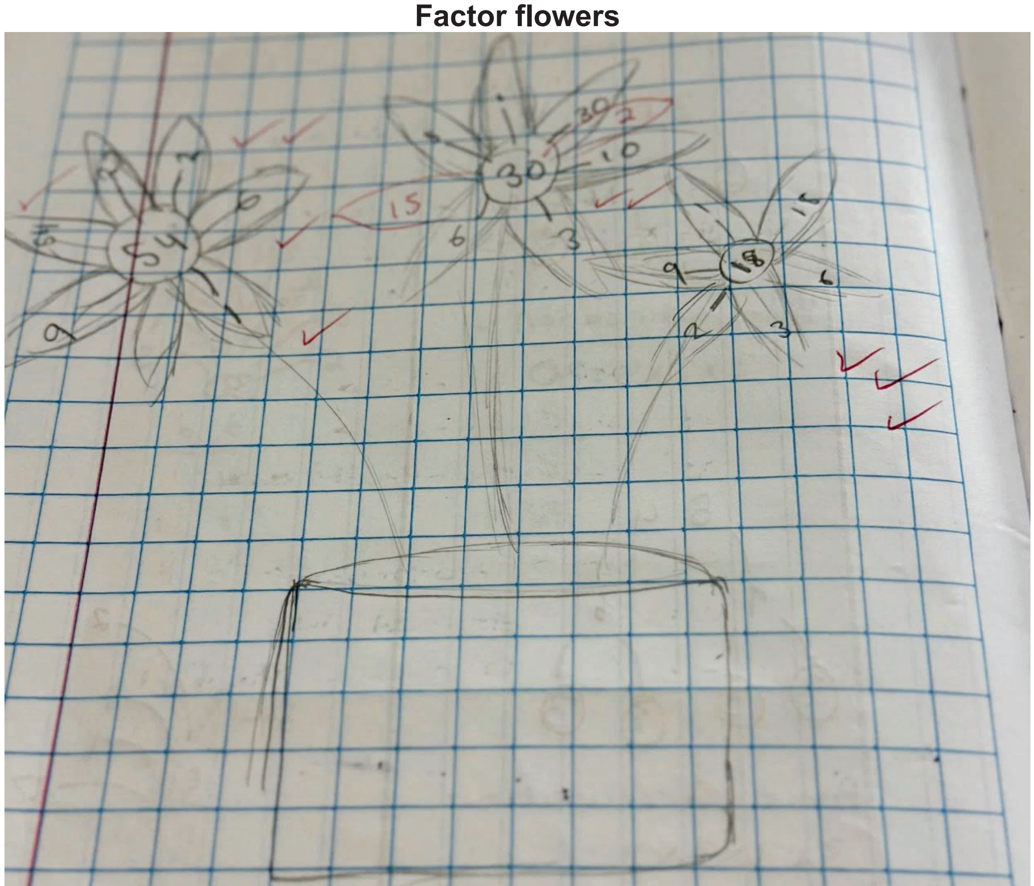
Alternative recording option – Factor Rainbows



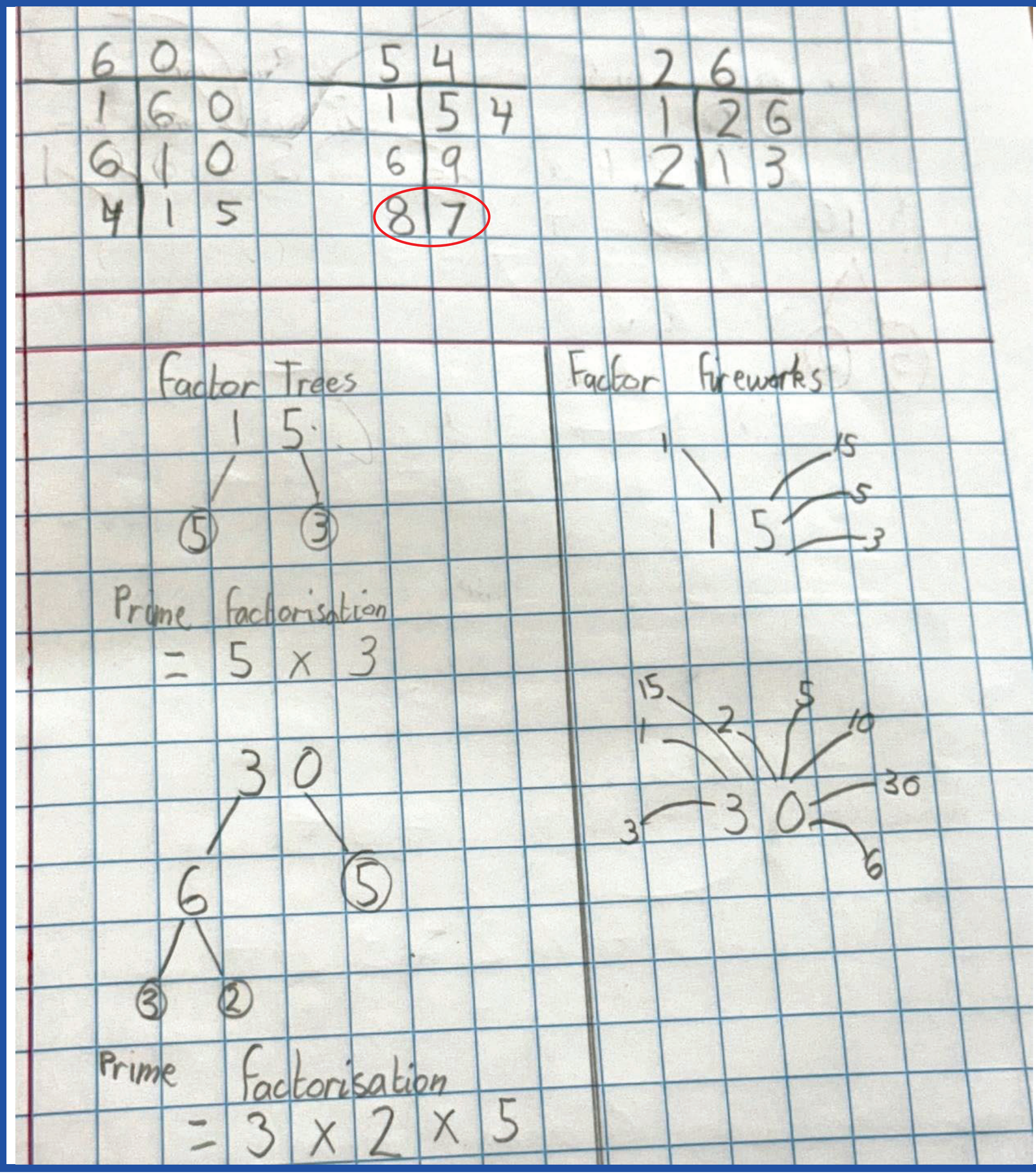
Alternative recording option – Factor Flowers

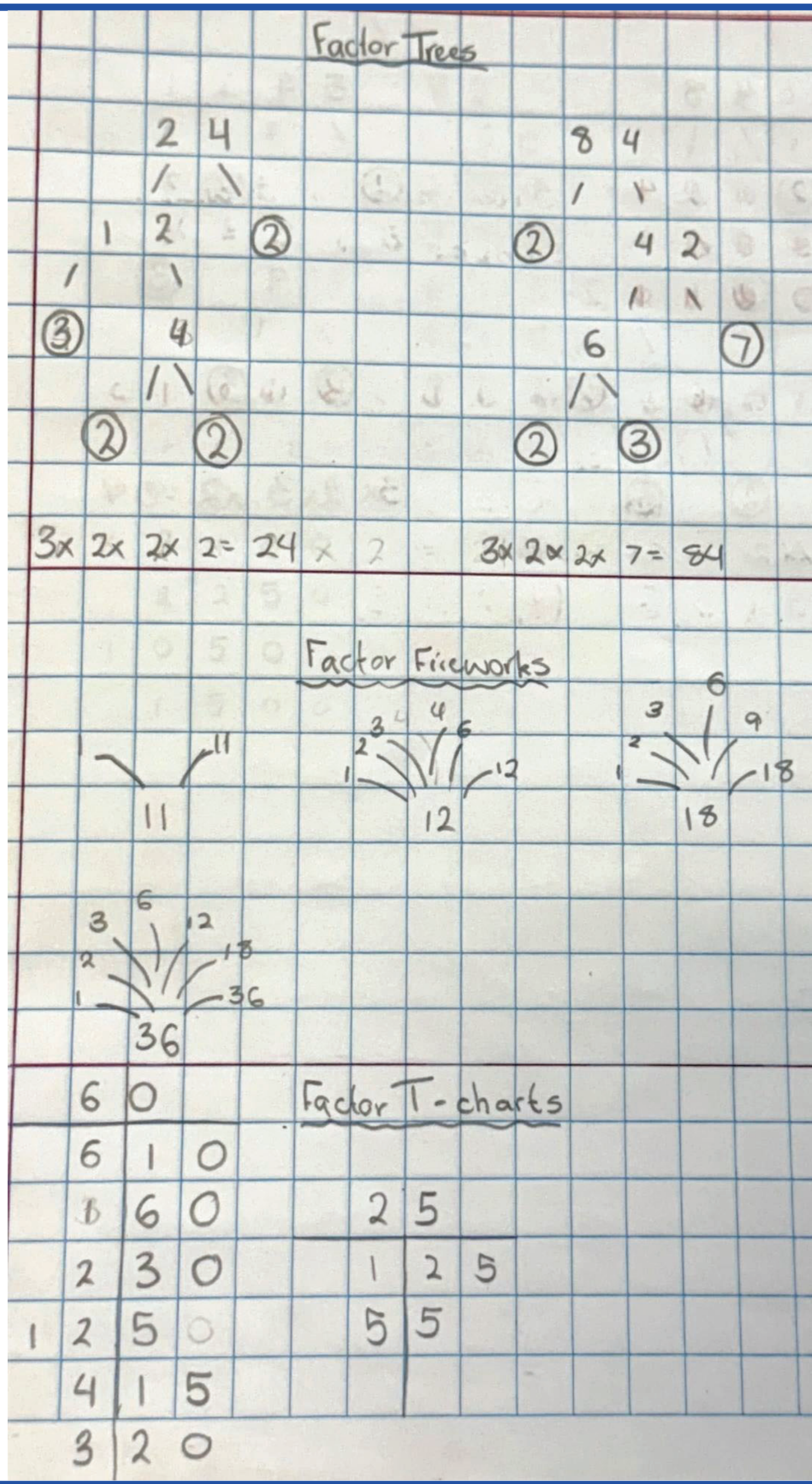


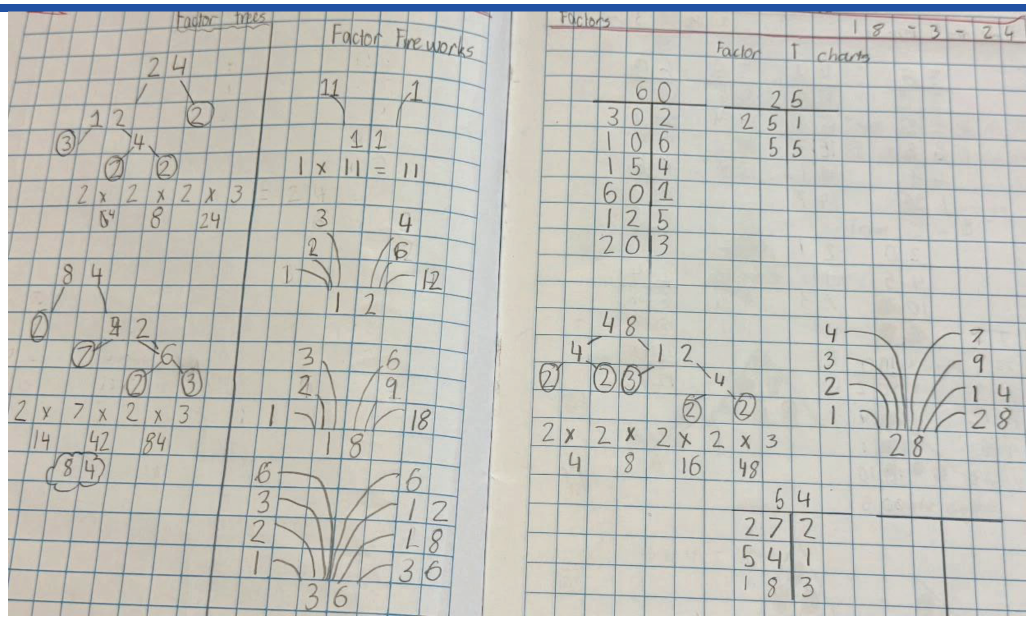
Factor flowers



Mix of recording




Factor Trees
24
84
3x 2x 2x 2= 24
3x 2x 2x 7= 84
Factor Fireworks
11
12
18
36
Factor T-charts
60
25



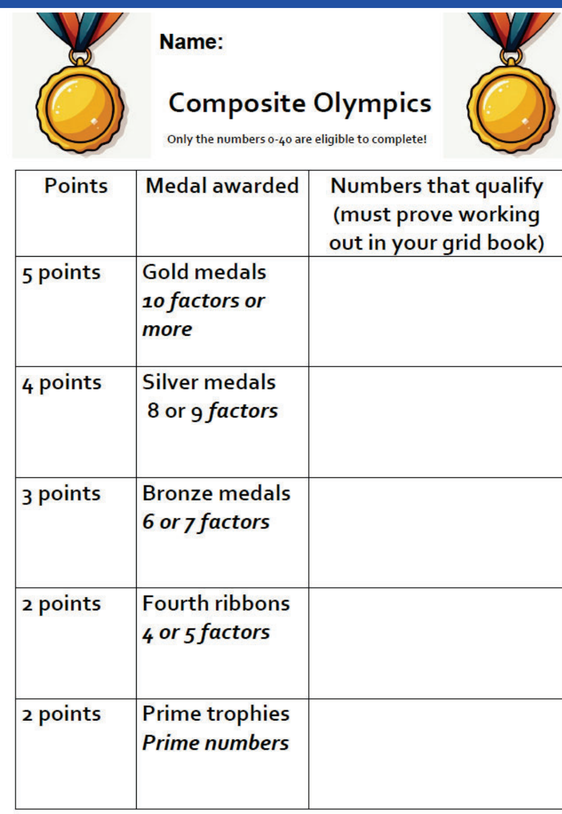
Variety of recording methods in use

Support adaptions: Use the **[support composite medal scoring template]**, which is more supportive in its points allocations, making the competition fair if played against mixed-ability peers.

Use counters to carry out the prime/composite test, testing whether the multiple can be made into an array (aside from one long row or one tall column).

Accordingly, decrease the limit from 120 to 40 to make arranging counters into arrays a manageable strategy.

Decrease the gold medal requirement from 12 factors to 10 (silver is then 8 or 9, bronze is 6 or 7).



Name:

Composite Olympics

Only the numbers 0-40 are eligible to complete!

Points	Medal awarded	Numbers that qualify (must prove working out in your grid book)
5 points	Gold medals *10 factors or more*	
4 points	Silver medals 8 or 9 *factors*	
3 points	Bronze medals *6 or 7 factors*	
2 points	Fourth ribbons *4 or 5 factors*	
2 points	Prime trophies *Prime numbers*	

Award 2 points for every prime number they locate up to 40 (rather than only 1 point, doubling the worth of the 'prime trophies' on the scoring sheet).

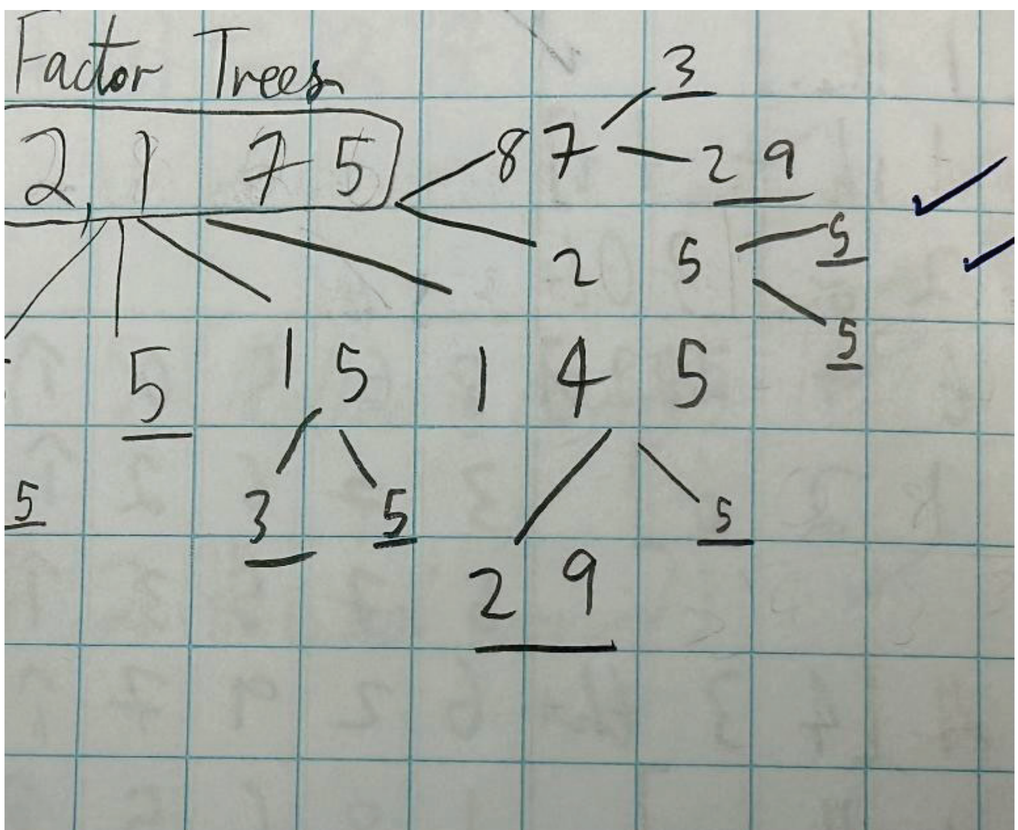
Extension 1: Remove the limit of 120 and increase it to 1000, or later 3000.

After these students earn most of the gold and silver medals, challenge them to find the most highly composite number they can. For example, can you find a number with 14 factors? 16? 20? There is one number that exists, below 1000, that has 32 factors, and another that has 30. Compete against an extension-level partner, aiming to find a more 'golden' composite number than them.

Tip for extension students who are struggling: Think about the numbers that were highly composite below 120. Will these numbers have a connection to the numbers that are highly composite above 120? If you multiply the gold medal numbers from below 120, see what totals you reach and investigate those multiples.

Answers:

180	18	1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60, 90, 180
240	20	1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 40, 48, 60, 80, 120, 240
360	24	1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360
720	30	1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 30, 36, 40, 45, 48, 60, 72, 80, 90, 120, 144, 180, 240, 360, 720
840	32	1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 14, 15, 20, 21, 24, 28, 30, 35, 40, 42, 56, 60, 70, 84, 105, 120, 140, 168, 210, 280, 420, 840



The highly composite number: 10080 10080 = (2 × 2 × 2 × 2 × 2) × (3 × 3) × 5 × 7					
1 × **10080**	**2** × **5040**	3 × 3360	**4** × **2520**	5 × 2016	**6** × **1680**
7 × 1440	8 × **1260**	9 × 1120	10 × 1008	**12** × **840**	14 × **720**
15 × 672	16 × 630	18 × 560	20 × 504	21 × 480	**24** × 420
28 × **360**	30 × 336	32 × 315	35 × 288	**36** × 280	40 × 252
42 × **240**	45 × 224	**48** × 210	56 × **180**	**60** × 168	63 × 160
70 × 144	72 × 140	80 × 126	84 × **120**	90 × 112	96 × 105
Note: Numbers in **bold** are themselves **highly composite numbers**.					

Extension 2: Investigate 10 080, a well-known highly composite number. How many factors does it have?

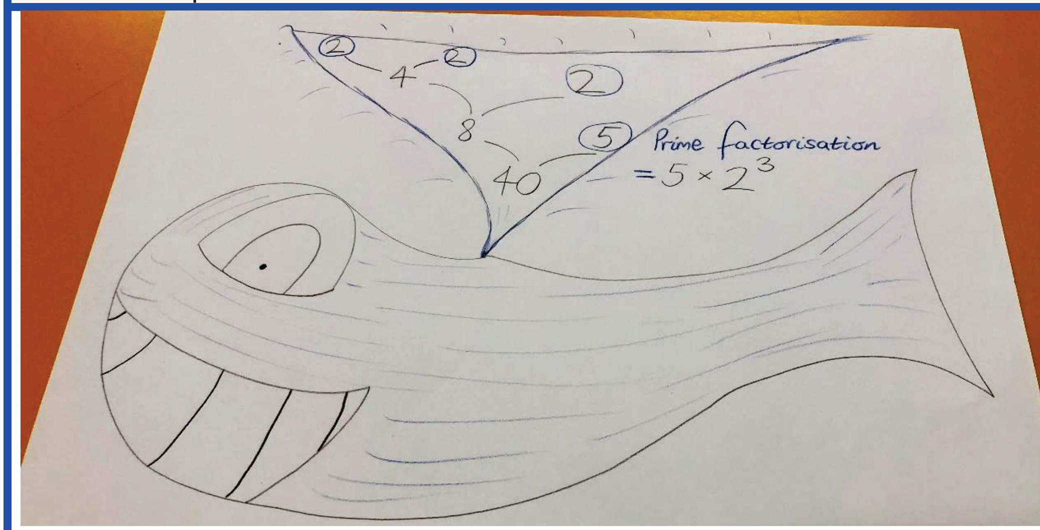
Reflection challenge

Question 1: How can you test whether a number is prime or composite with fair confidence? What sort tests would you do first and in what order?

Question 2: What is a great strategy to find highly composite numbers? Multiply smaller highly composite numbers by 12, or 2, 4.

NRICH follow-on option

Factors and multiples puzzle:
https://nrich.maths.org/problems/factors-and-multiples-puzzle

Prime Composite Square Triangular Year 6C Lesson 3	**Prime Factor Artwork** **Learning intention: Use factor trees to identify the prime factors of any multiple. Differentiate between composite and prime factors. Use the double it halve it factor strategy. Record numbers in their prime factorisation (as a product of their prime factors).** **Maths vocabulary: prime factorisation (prime factors in index/exponential notation, known informally as the prime fingerprint), factor trees, product, prime factor, composite factor, fundamental theorem of arithmetic, commutative law**
Link to the arts: Go on a virtual tour of some recent exhibits https://www.ngv.vic.gov.au/virtual-tours/ and a list of others available at https://www.travelandleisure.com/attractions/museums-galleries/museums-with-virtual-tours	**Lesson summary: Students create factor trees that continue to branch down until all prime factors of the multiple have been located, then circle these prime factors and record the prime factorisation of that number using index notation (Year 7 standard).** **Students publish their favourite factor tree as a factor art gallery, emphasising that the number will always have the same prime factors, regardless of how the factor tree starts.** **Materials:** • Grid books for drafting their factor trees and practising prime factorisation. • Teacher modelled examples of factor artwork (or use the student work samples copied in the images within this lesson plan). • A3 paper for publishing students' chosen pieces. **Best set-up:** Show students examples from the images on the following pages. Set students to work drafting factor trees and prime factorisations in their grid books, aiming to create one that they are most proud of to publish as their final piece of 'factor art' at the end of the session.  **Factor art student work sample (published version after drafting many factor trees and prime factorisations in their grid book)**

Modelling: For this lesson, students start by choosing some of the 'gold' and 'silver' medal **highly composite** numbers from the previous lesson.

Students then draw a factor tree from this total, but instead of stopping after one level of branches (like the last session), students continue to branch out the composite factors until the end of each branch reaches a prime factor.

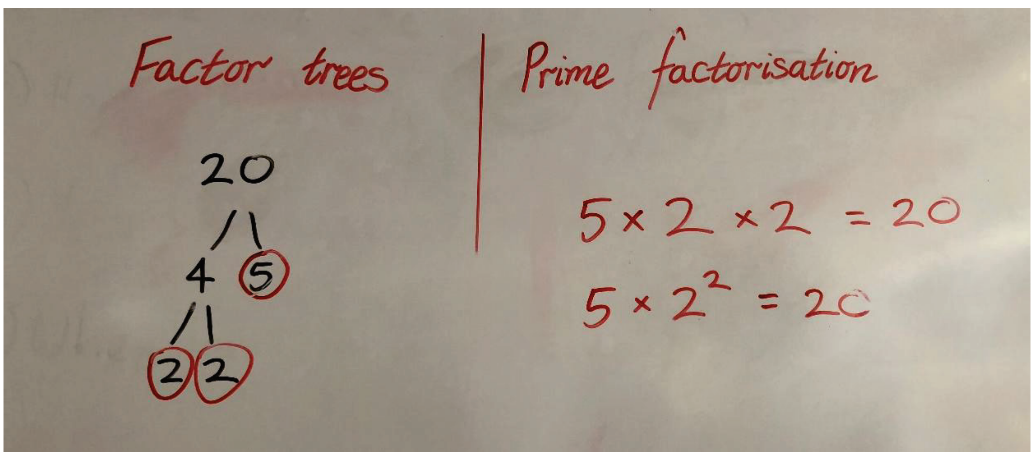
A **composite factor** is a number you can still break apart into two factors (except for 1 and itself). A prime factor is essentially the end of the road, as the factors have been broken down as much as possible and it would just keep breaking into 1 and itself from then on.

Prime factorisation: After reaching all the prime factors in that factor tree, circle these prime numbers in red (like a stop sign, because the factor trees stops with these). Write all the circled prime numbers into a multiplication number sentence.

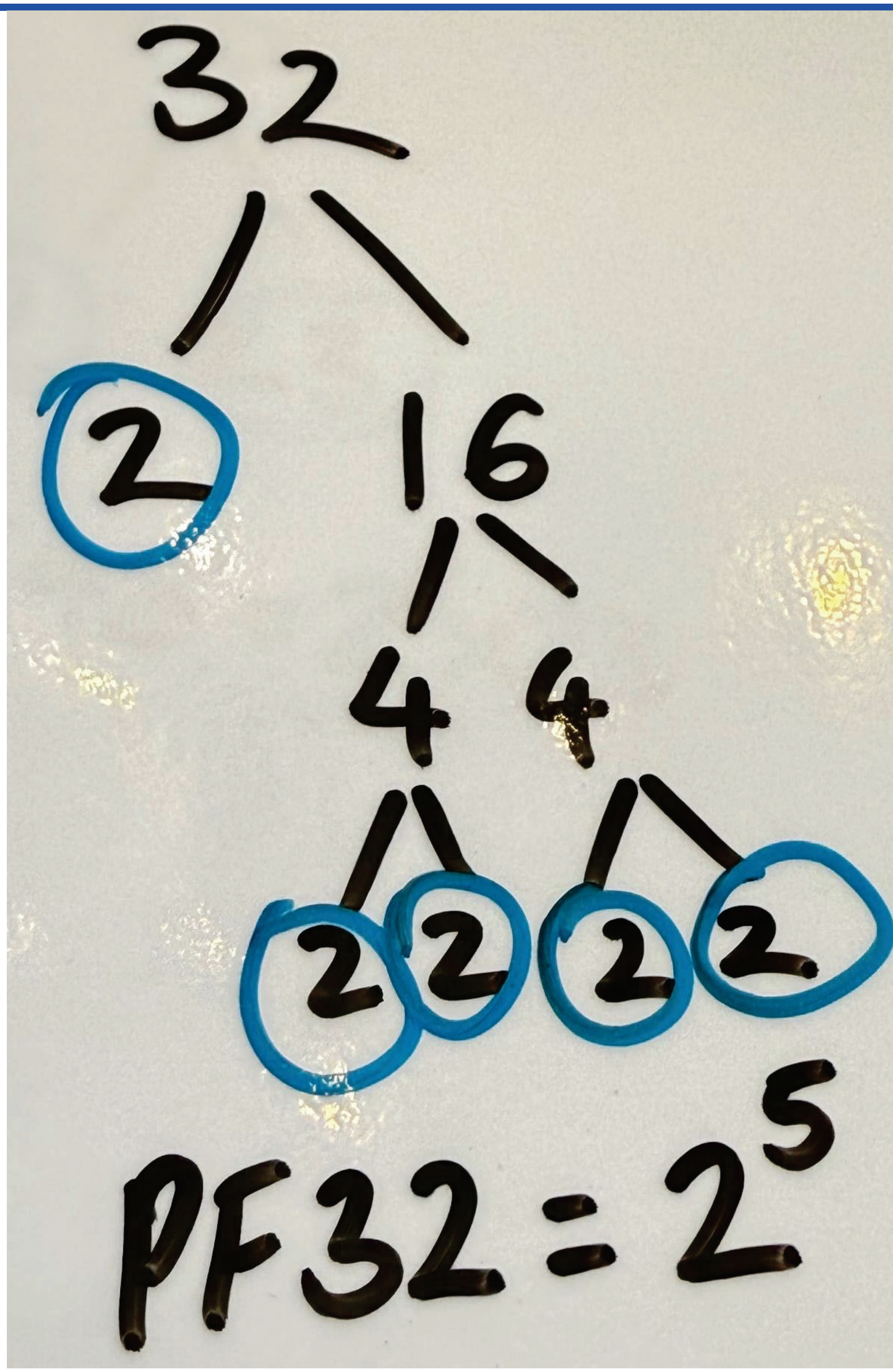
Index notation: Group the numbers that are the same and write them in index notation. For the prime factorisation in index notation, if the factors are 2 x 2 x 2 x 5 for example, it would be: PF = 2^3 x 5 (because 2 has been multiplied by itself 3 times).

Misconception alert: Highlight that 2^3 is not 2 x 3, but 2 x 2 x 2 (2 multiplied by itself three times). At first, draft many examples in their books, for at least one whole session, if not two sessions.

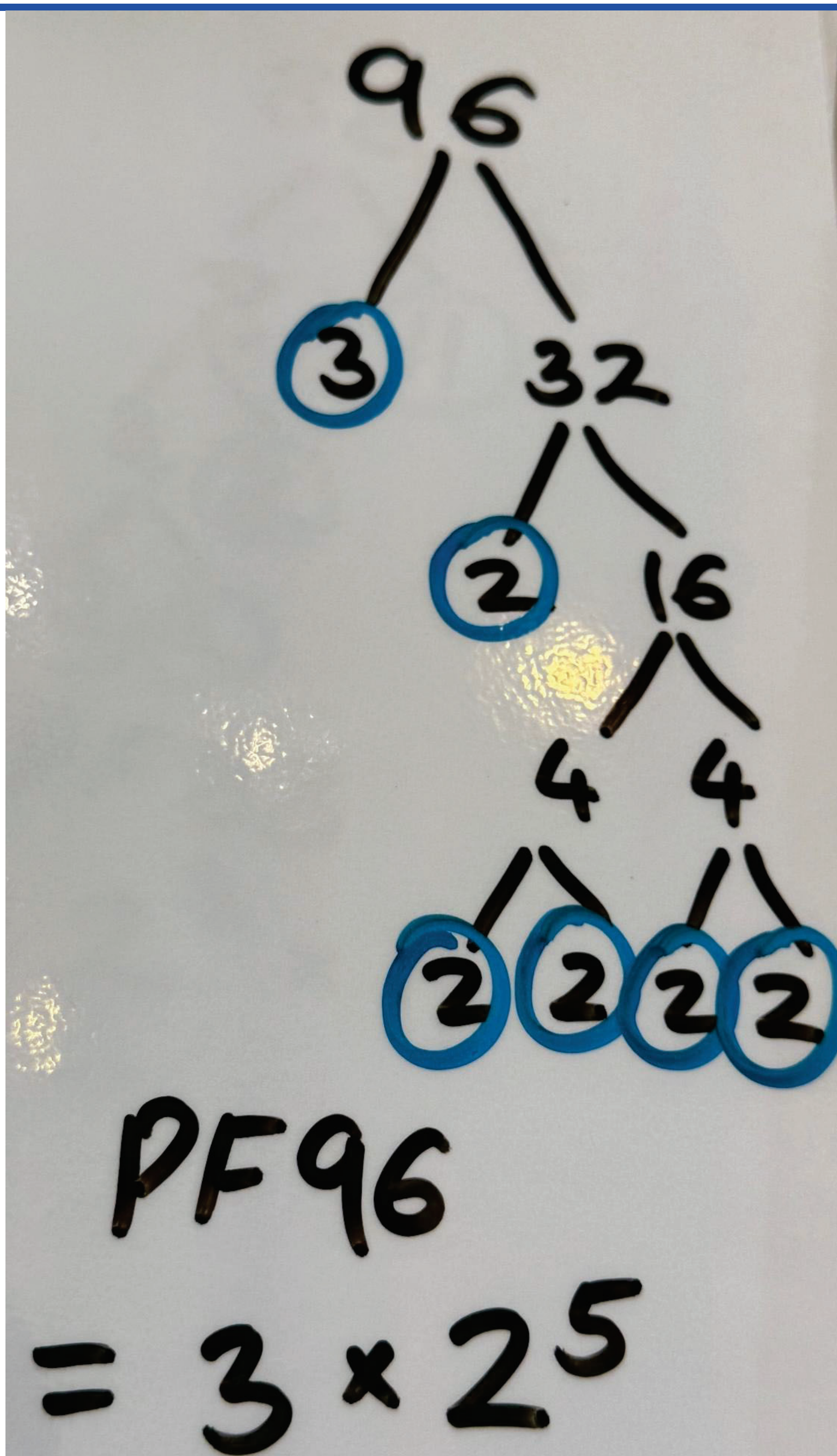
Website with worked examples:
https://www.cuemath.com/numbers/factors-of-64/ (scroll down to the factor tree example of 64) and https://www.cuemath.com/numbers/factors-of-80/.



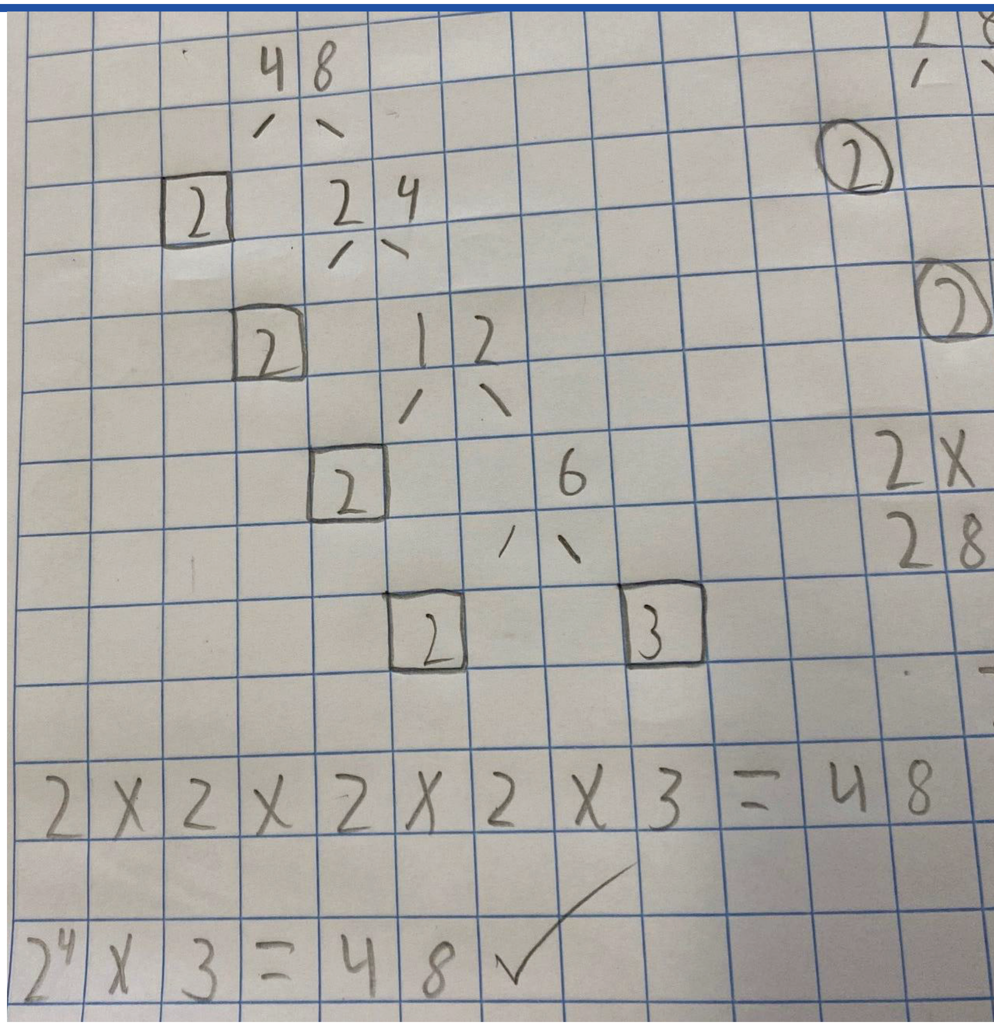
Initial whiteboard modelling

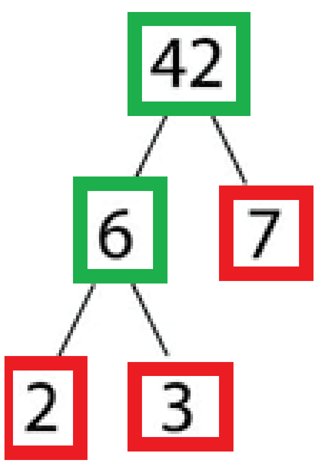


Initial whiteboard modelling where 'PF' stands for 'Prime factorisation'



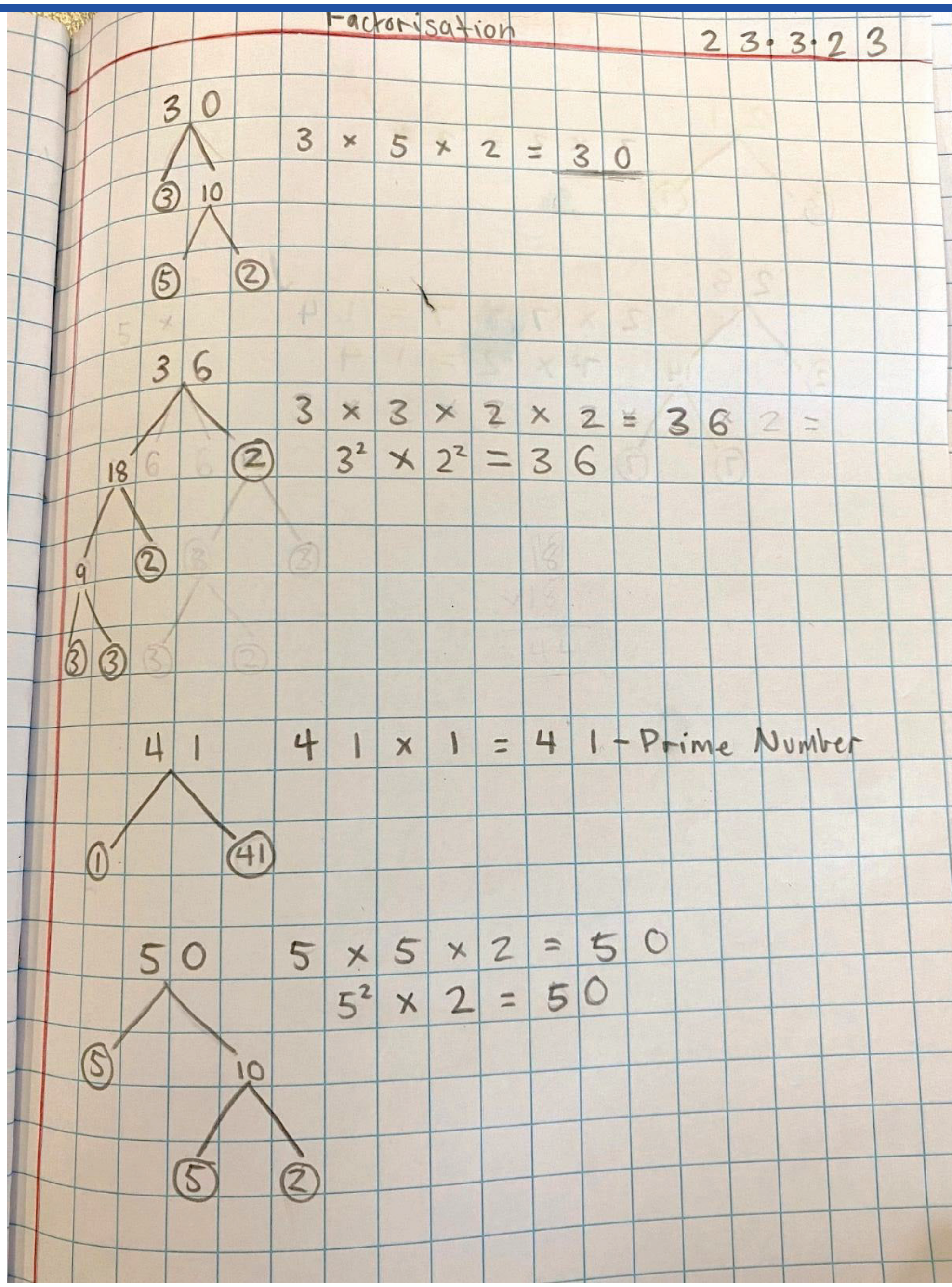
96
3
32
2
16
4
4
2
2
2
2
PF96
= 3 × 2⁵



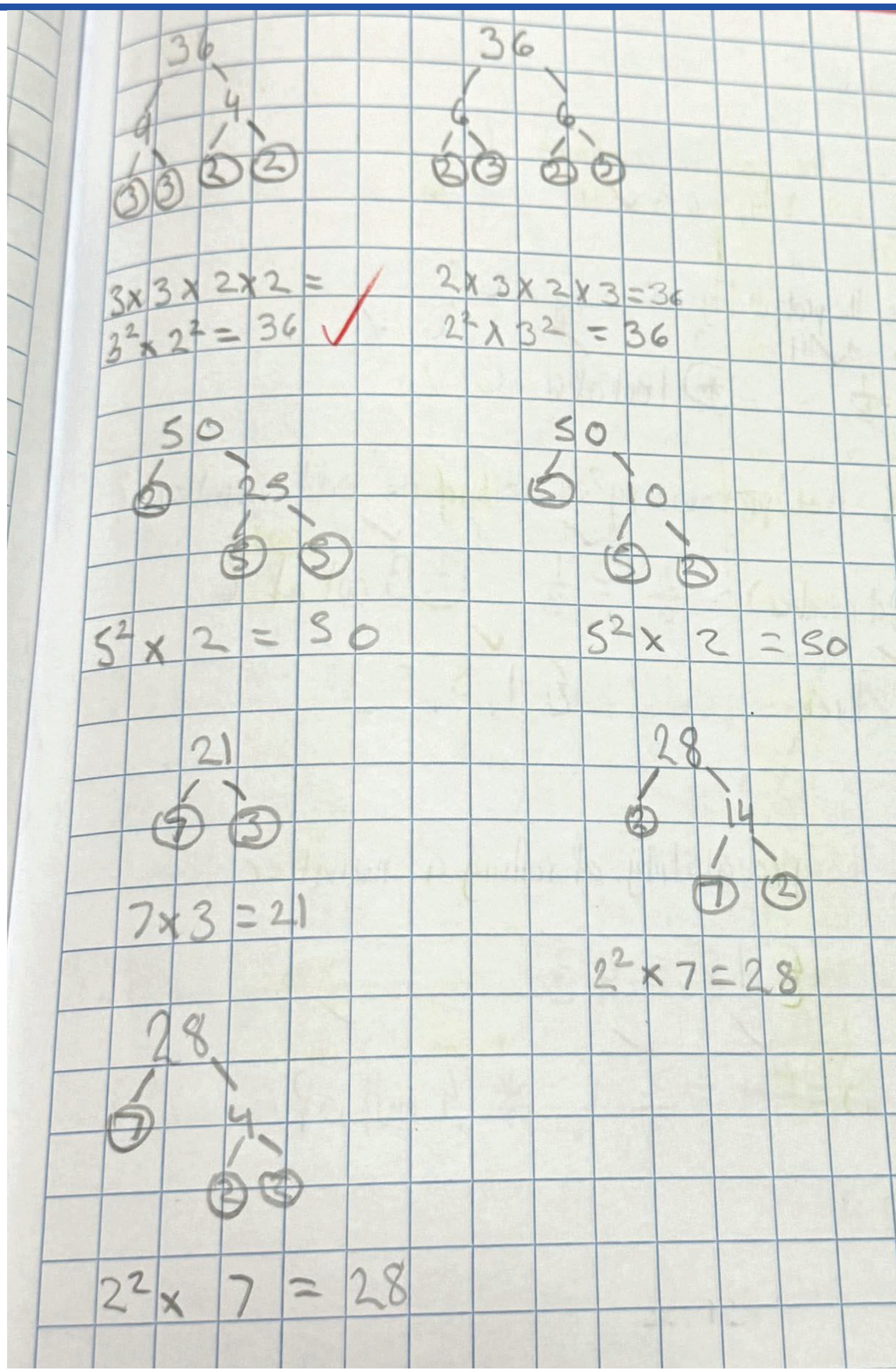


Some factors of 42: 6, 7, 2, 3
Prime factors: 2, 3 and 7

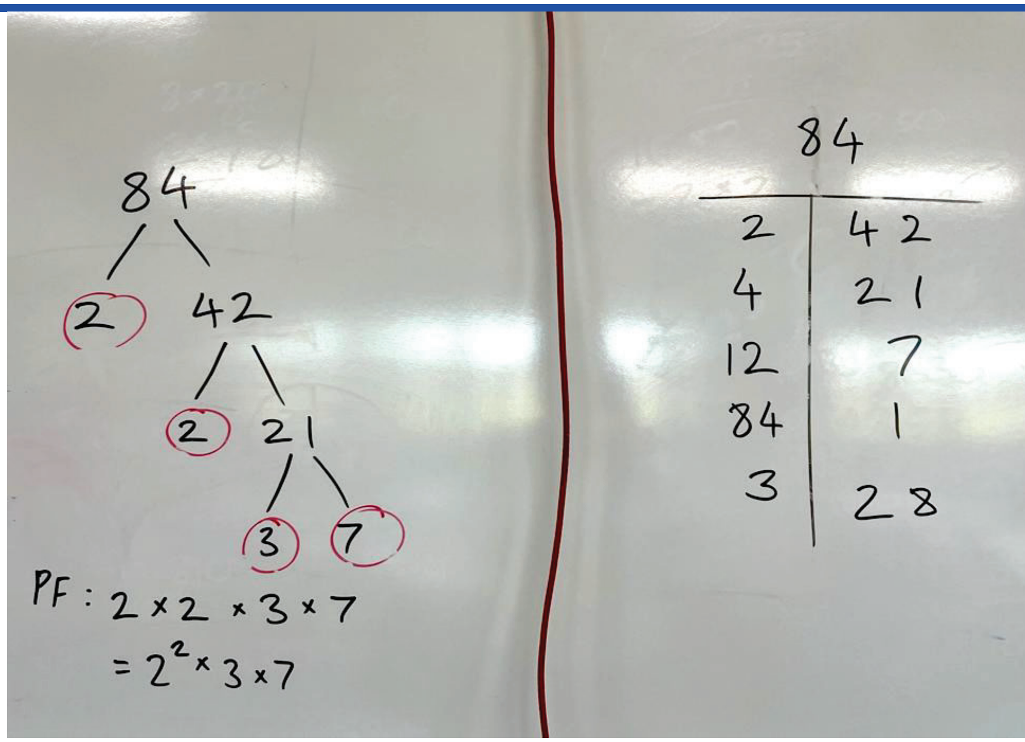
Prime factorisation = 2 x 3 x 7

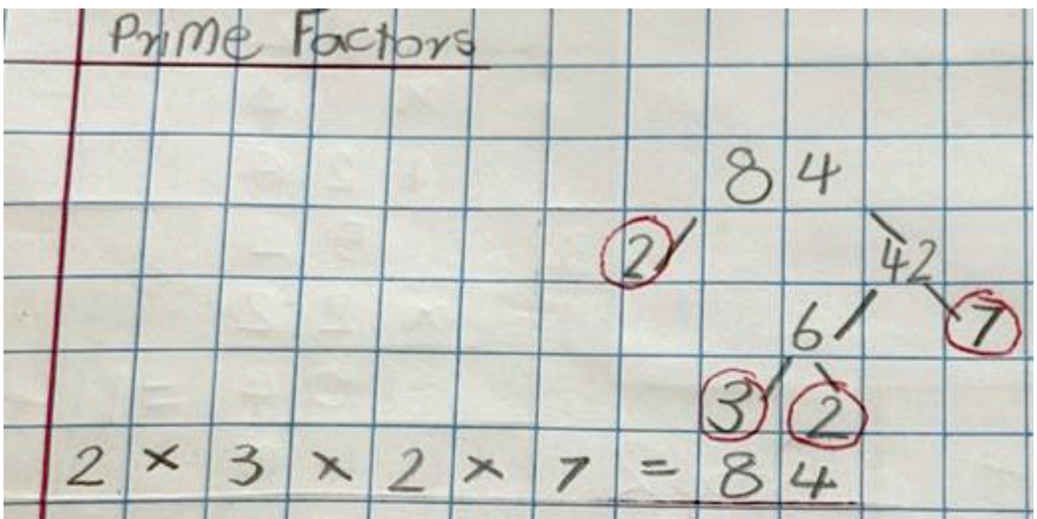


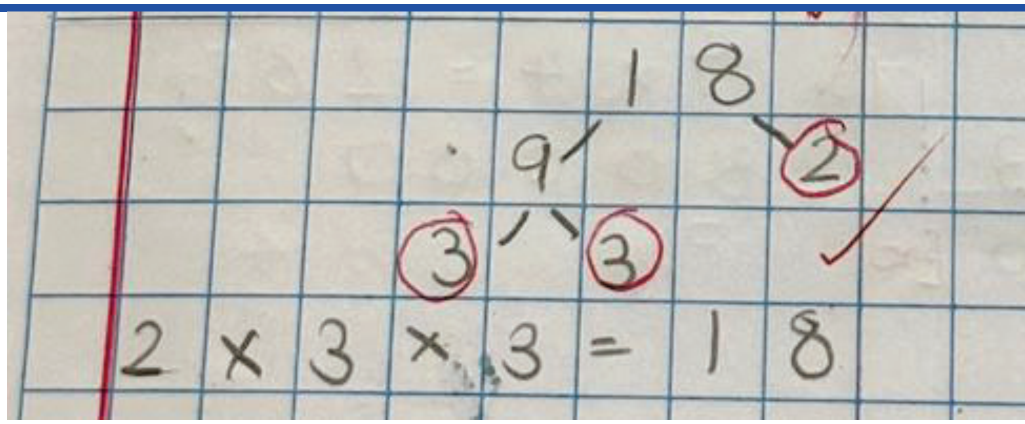
Chirnside Park PS student work sample

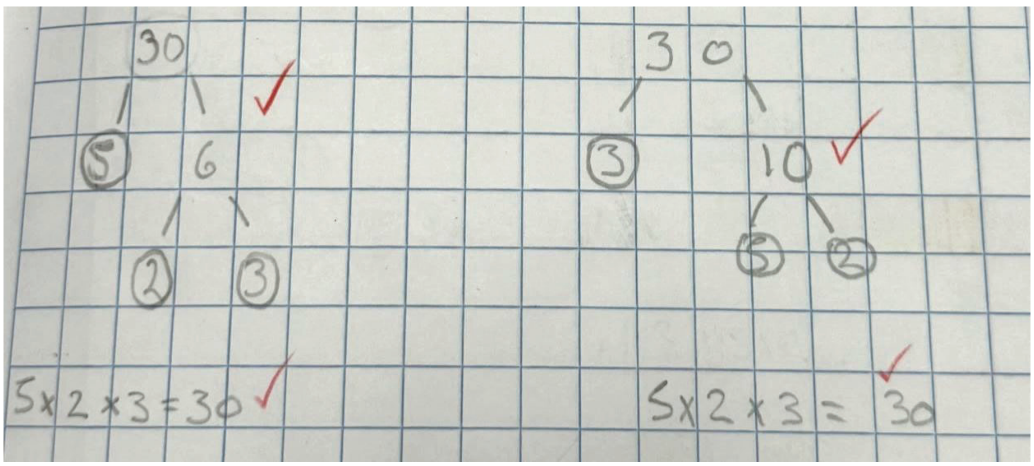


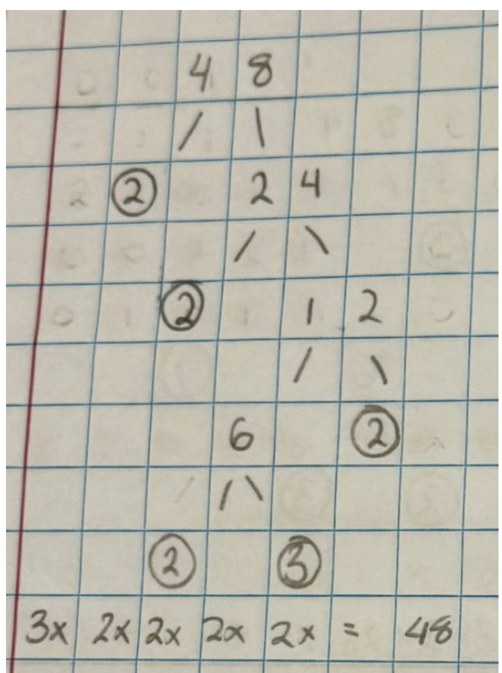
Creating the prime factorisation, or prime fingerprints, by using factor trees and circling all the prime factors at the end of each branch.

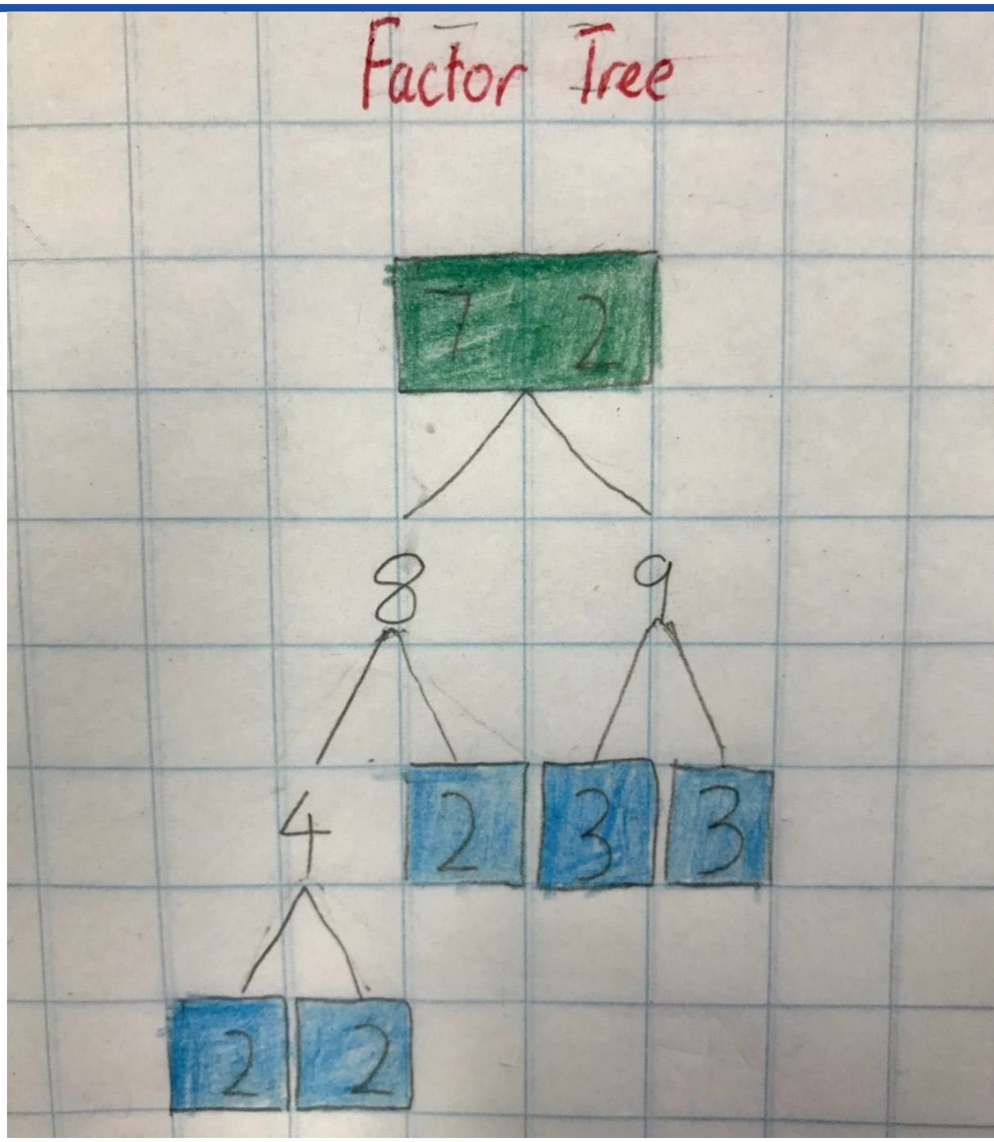

84
2
42
2
21
3
7
PF : 2 × 2 × 3 × 7
= 2² × 3 × 7
84
2 42
4 21
12 7
84 1
3 28

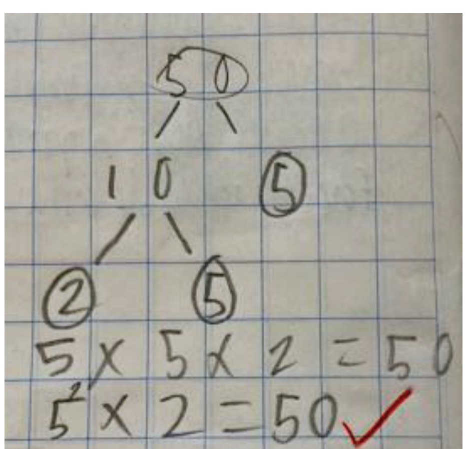

Prime Factors
84
2
42
6
7
3
2
2 × 3 × 2 × 7 = 84

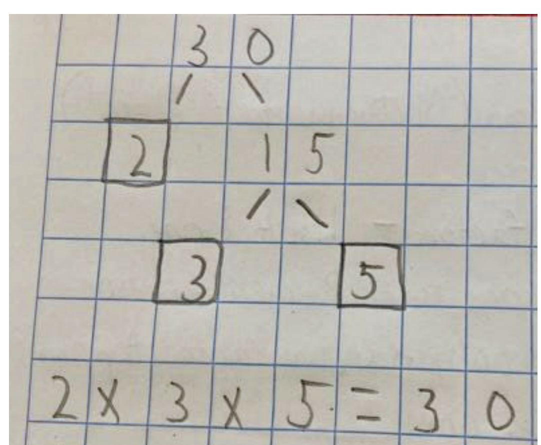

18
9
2
3
3
2 x 3 x 3 = 18

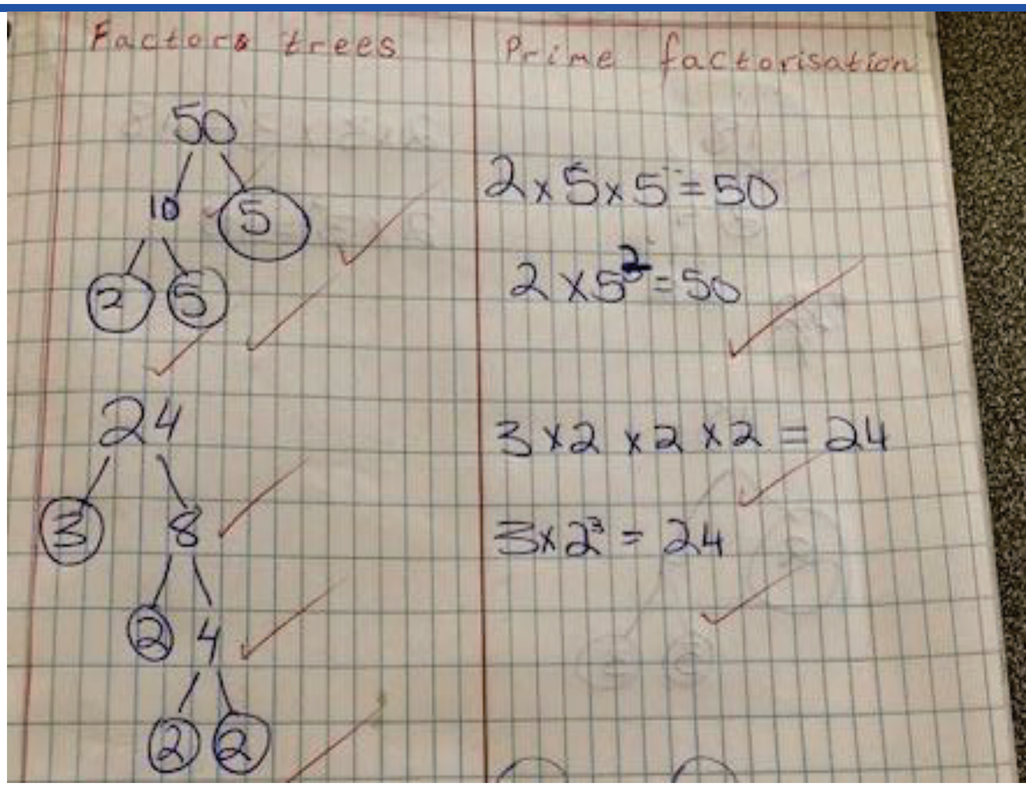

30
5
6
2
3
5x2x3=30
30
3
10
5
2
5x2x3 = 30


48
2
24
2
12
6
2
2
3
3x 2x 2x 2x 2x = 48


Factor Tree
72
8
9
4
2
3
3
2
2


50
10
5
2
5
5 x 5 x 2 = 50
5² x 2 = 50

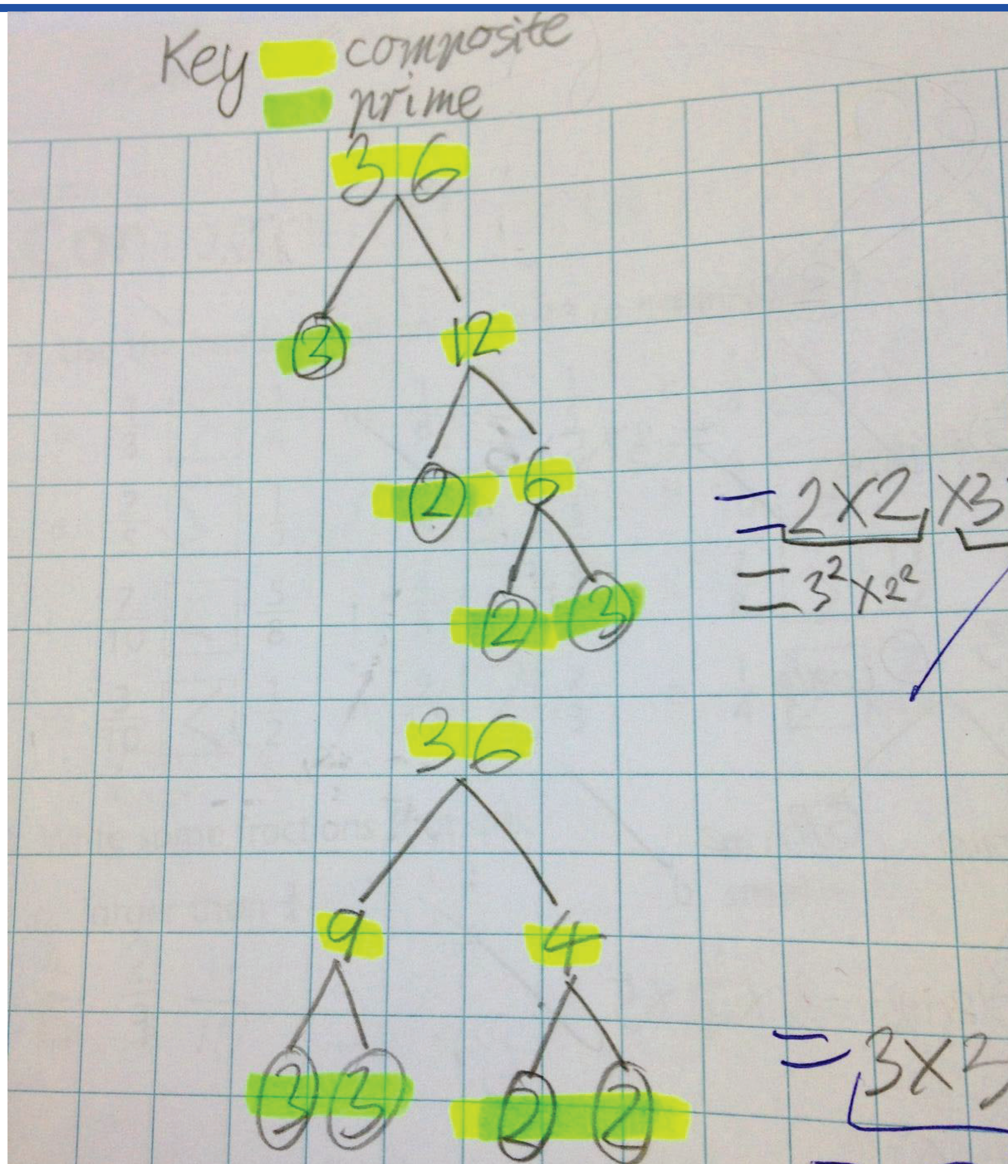

30
2
15
3
5
2 x 3 x 5 = 30

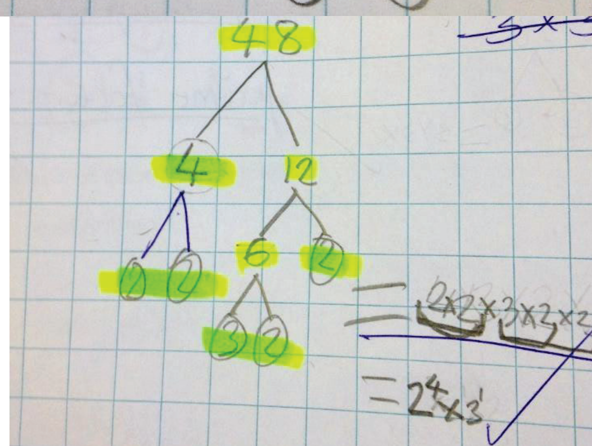


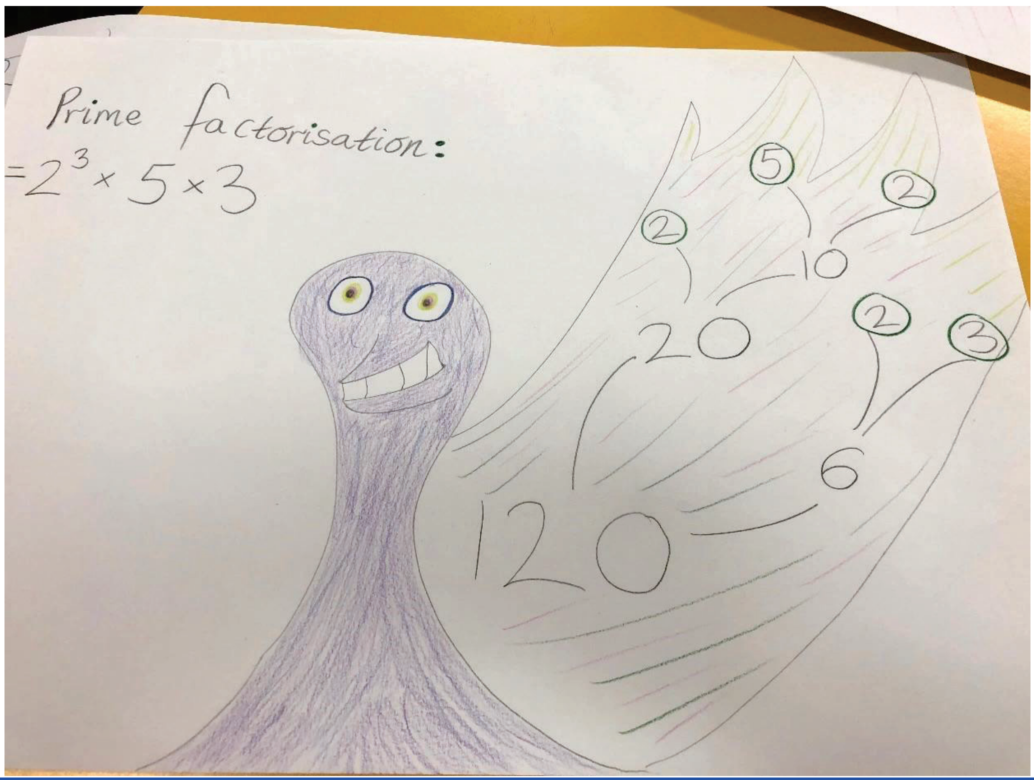
Prime fingerprints: Restart factor trees a few times from the same total, aiming to make a few different factor trees from that multiple. Do the prime factors change or are they always the same? This is why prime factors are also referred to as the number's **prime fingerprint.** No two numbers shared the same set of prime factors, or the same prime fingerprint. It is like our fingerprints as humans, they are unique to us alone. This is formally known as the **Fundamental Theorem of Arithmetic**. It is also due to the **commutative law** – you can mix up the order of the factors in the number sentence, but it will always make the same **product** (total).

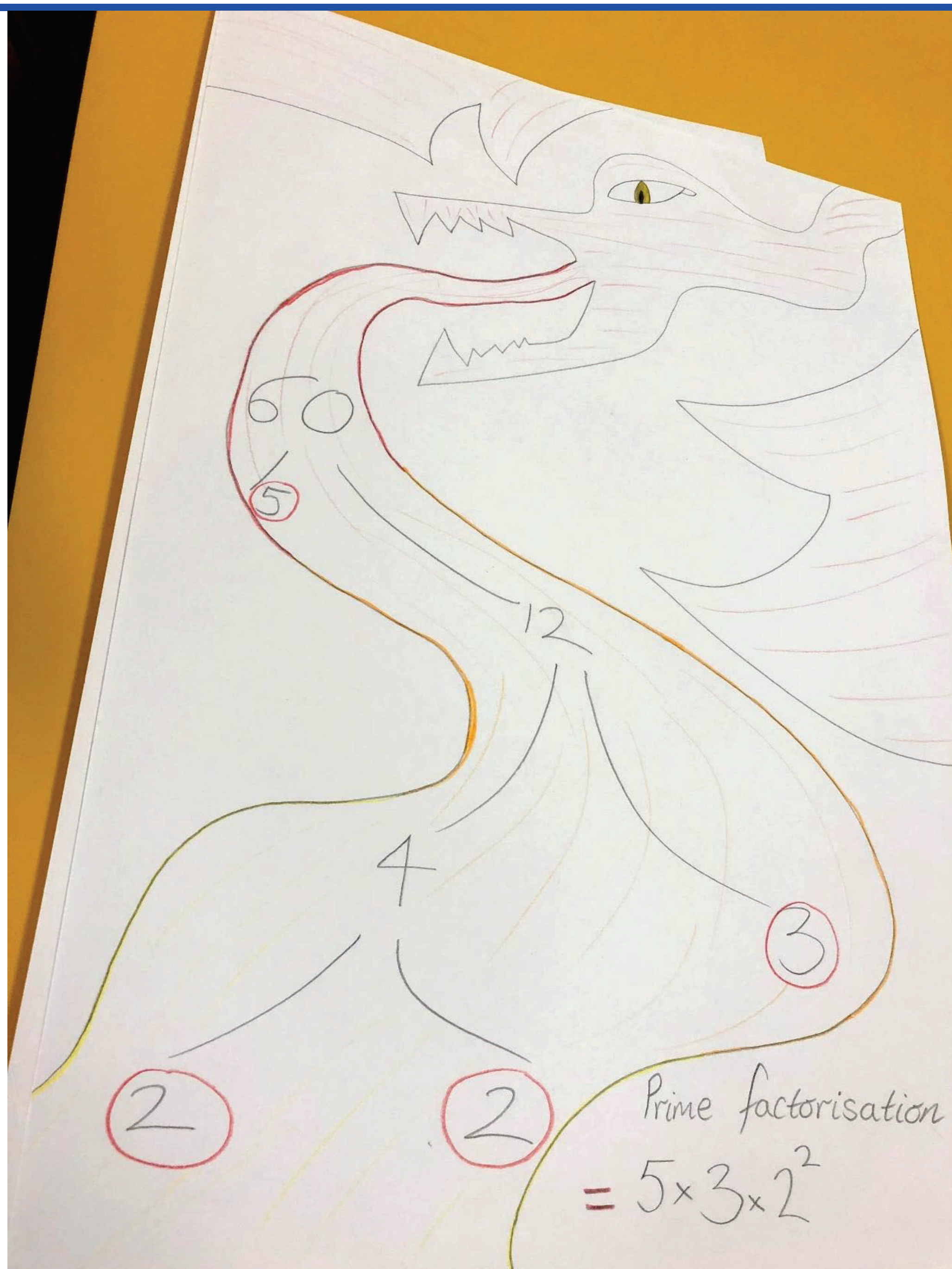
Misconception alert: Ensure students do not mix up adding and multiplying in the factor trees. The numbers in factor trees, since they relate to factors and ultimately arrays, are always multiplied together – not added.

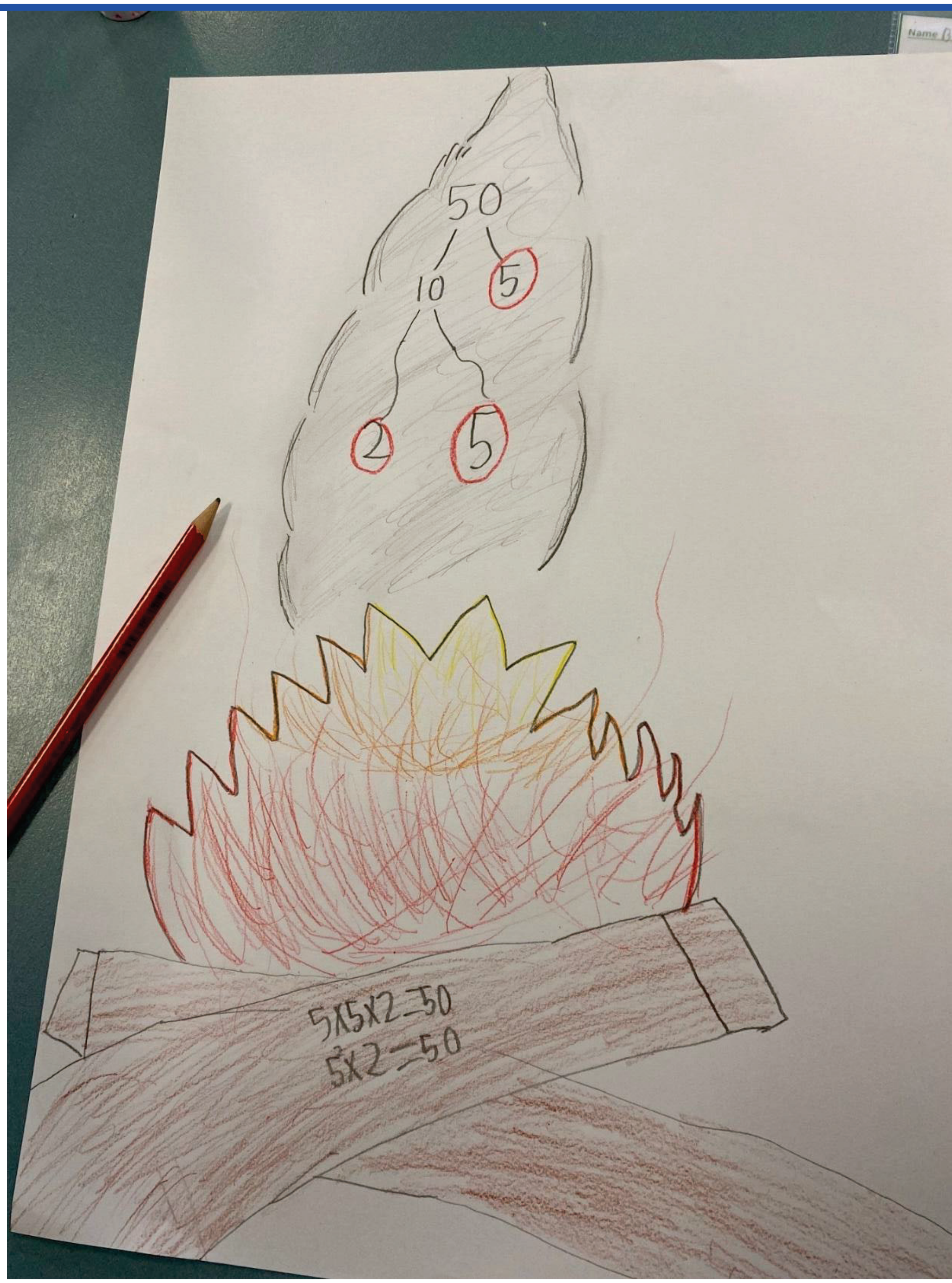
Prime numbers: Try to make a factor tree for a prime number. Is it interesting or very boring? What is the prime factorisation of any prime number? Just itself x 1.


Key
composite
prime
36
3
12
2
6
2
3
= 2x2x3
= 3²x2²
36
9
4
3
3
2
2
= 3x3

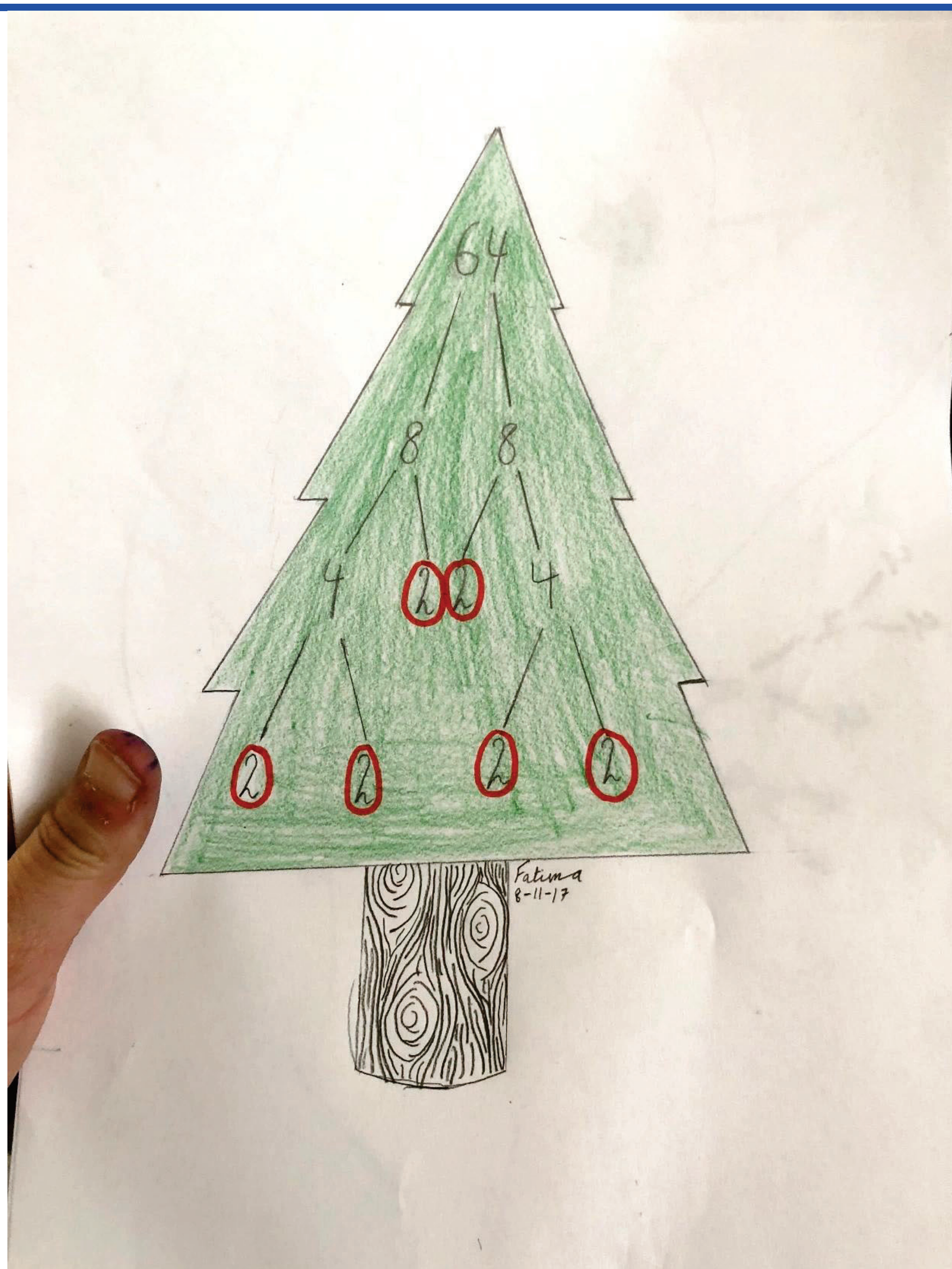

48
4
12
2
2
6
2
3
2
= 2x2x3x2x2
= 2⁴x3¹

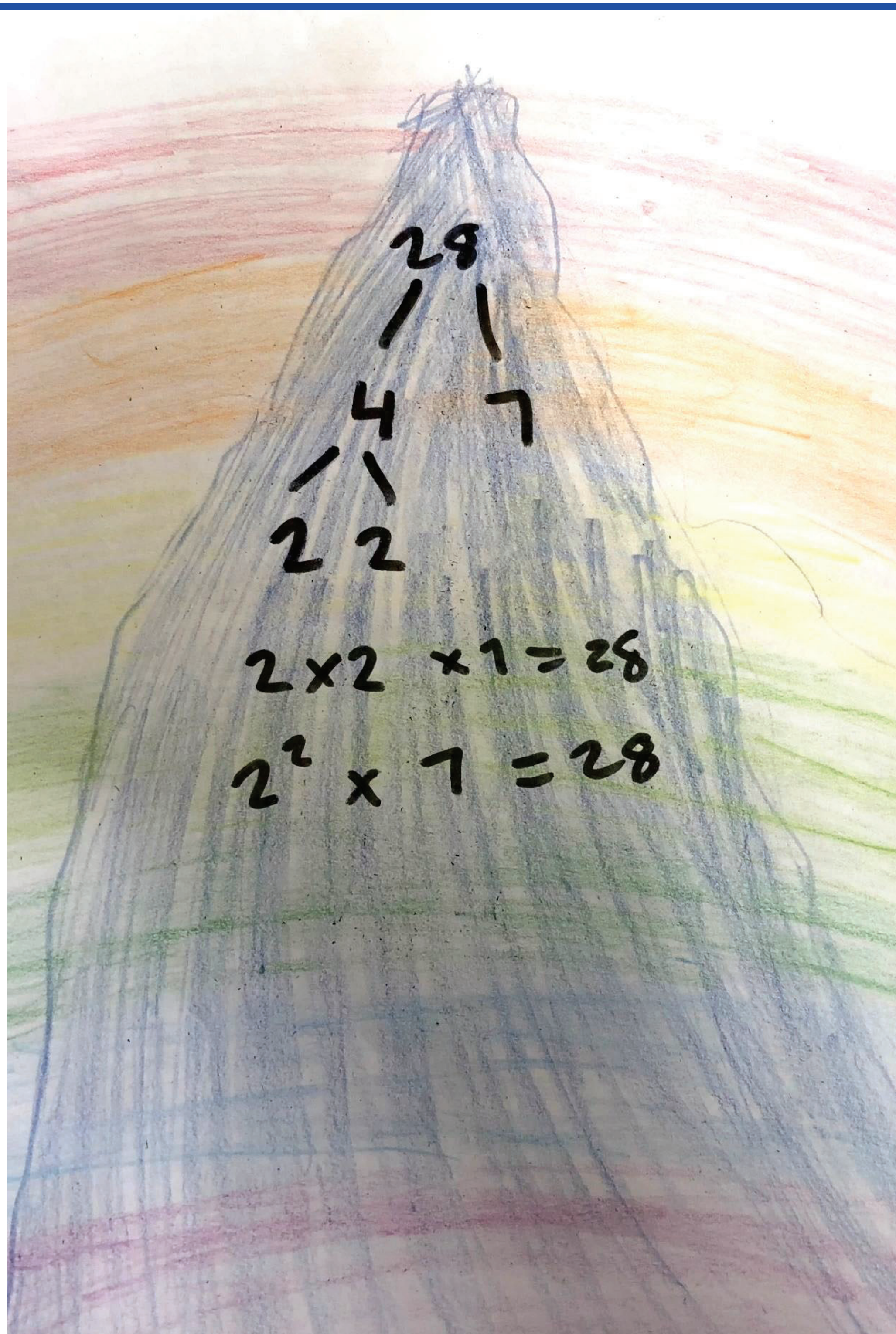
	Publishing art galleries: After students have investigated many different multiples, choose their favourite and choose an artistic context for their work. For example, a whale spurting out its factors from its blowhole, or a peacock with factors exploding from its plumage, a drink bottle squirting factors through its lid – anything! Publish at least 3 different factor trees for that total (so that the first 2 factors in the tree start differently) but it shows that the prime factors at the end of the branches remain the same. Publish the prime factorisation in index form on their gallery as well. At the end, lay out their galleries on the desk and students then do a literal gallery walk to appreciate and admire the work of their peers. 

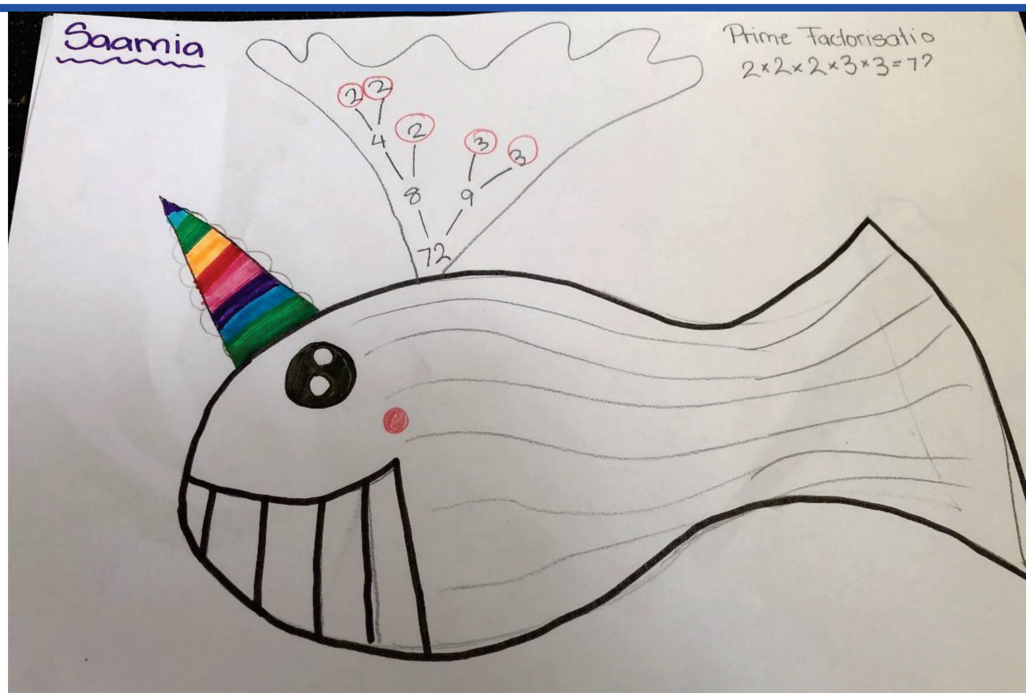

60
5
12
4
3
2
2
Prime factorisation
= 5×3×2²

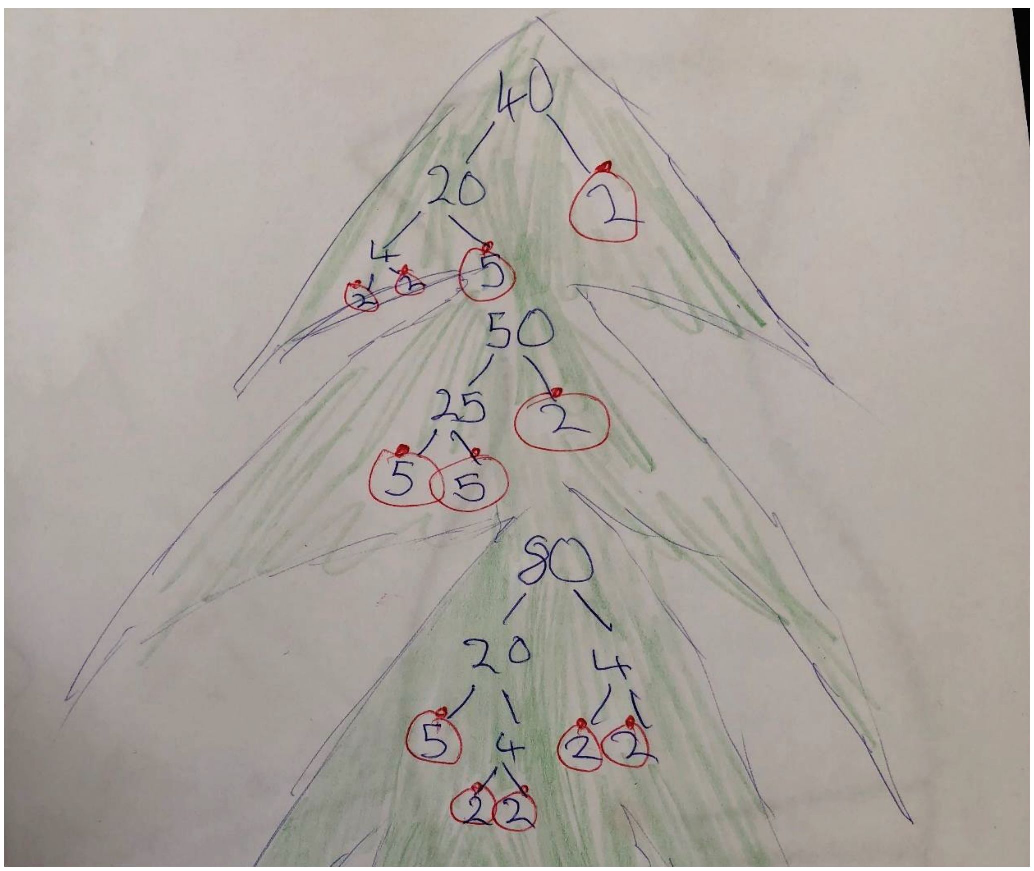


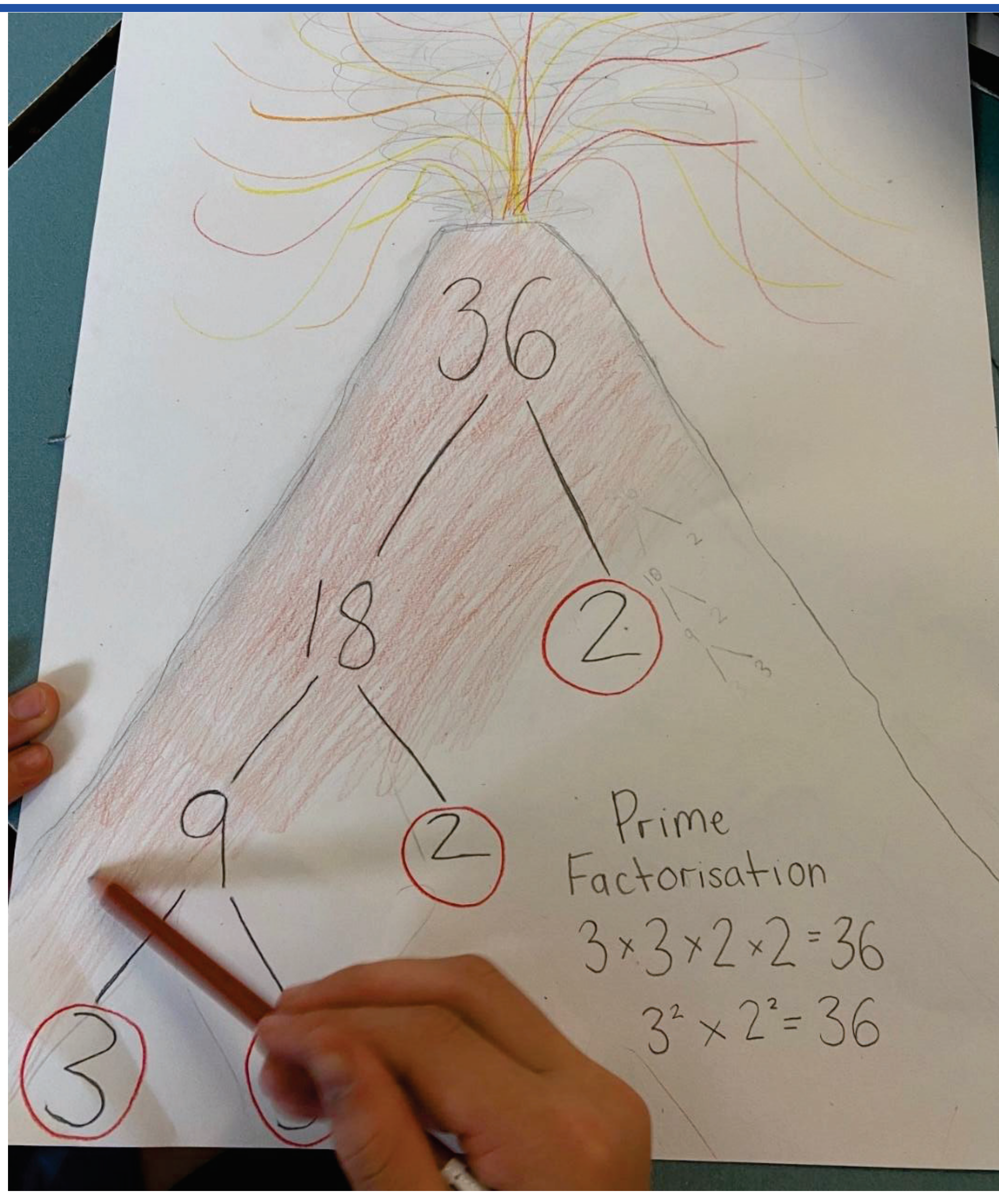
Chirnside Park PS student work sample


64
8
8
4
2
2
4
2
2
2
2
Fatima
8-11-17

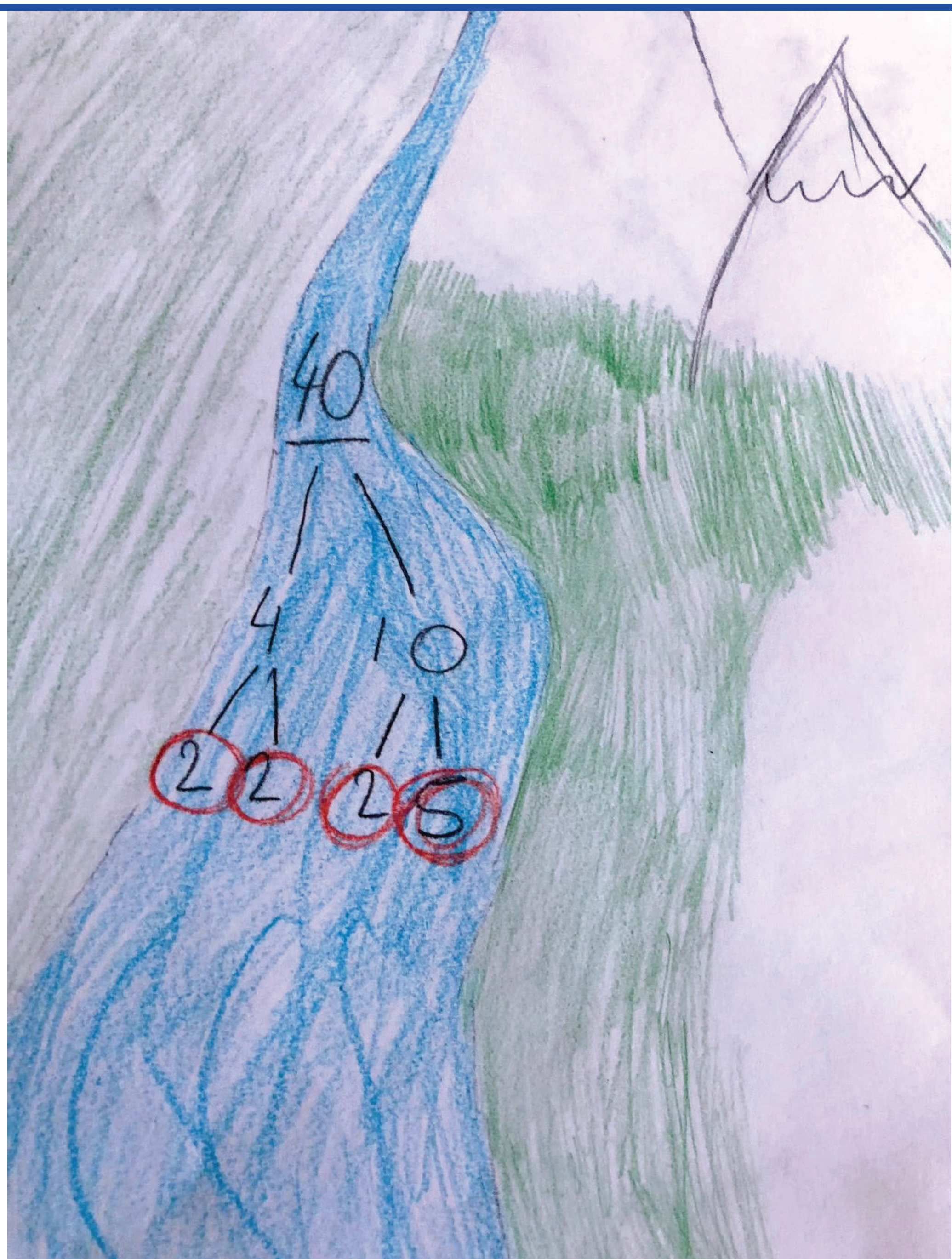

28
4 7
2 2
2x2 x7=28
$2^2 \times 7 = 28$


Saamia
Prime Factorisatio
2×2×2×3×3=72
72
8
9
4
2
3
3
2
2

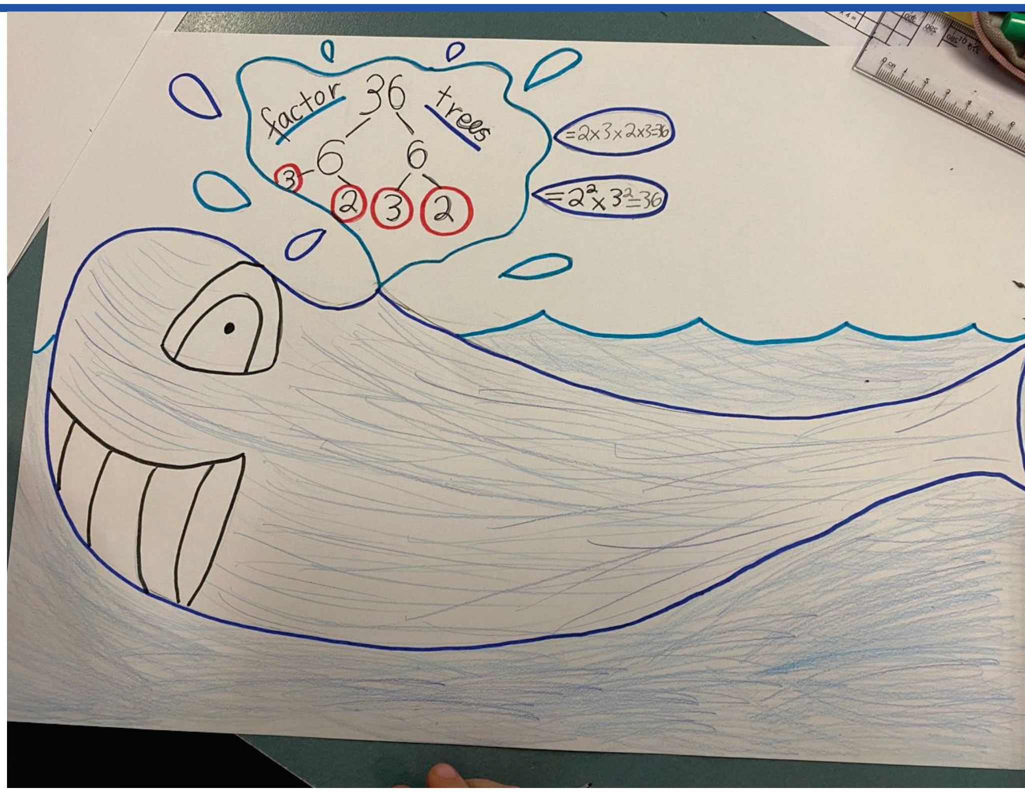

40
20
2
4
5
2
2
50
25
2
5
5
80
20
4
5
4
2
2
2
2



Chirnside Park PS student work sample


40
4
10
2
2
2
5


By: Lithurshigaa
Prime
Factorisation:
= 3 × 2 × 2² = 84
84
6 × 14
3 2 7 2



Chirnside Park PS student work sample

Support: Keep the starting total small and therefore more workable within their known times tables. Use counters to make arrays to solve the first few branches. Ignore the index notation – just write the multiplication sentence.

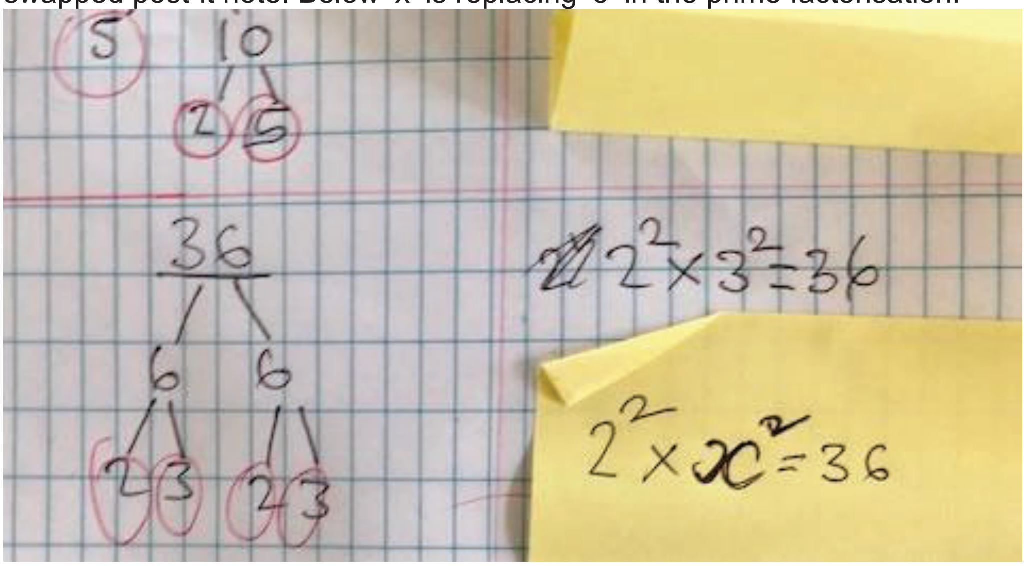
Support – extreme support: Just make single-tiered factor trees and keep earning gold, silver or bronze medals from the previous lesson. Publish the number they find that is the most highly composite as a piece of factor art.

Extension 1 – Problem-solving trickery: Tell extension students that you actually told the class a small fib. There are two numbers under 120 that actually do share the same set of prime factors (they have the exact same prime fingerprint). Find them!

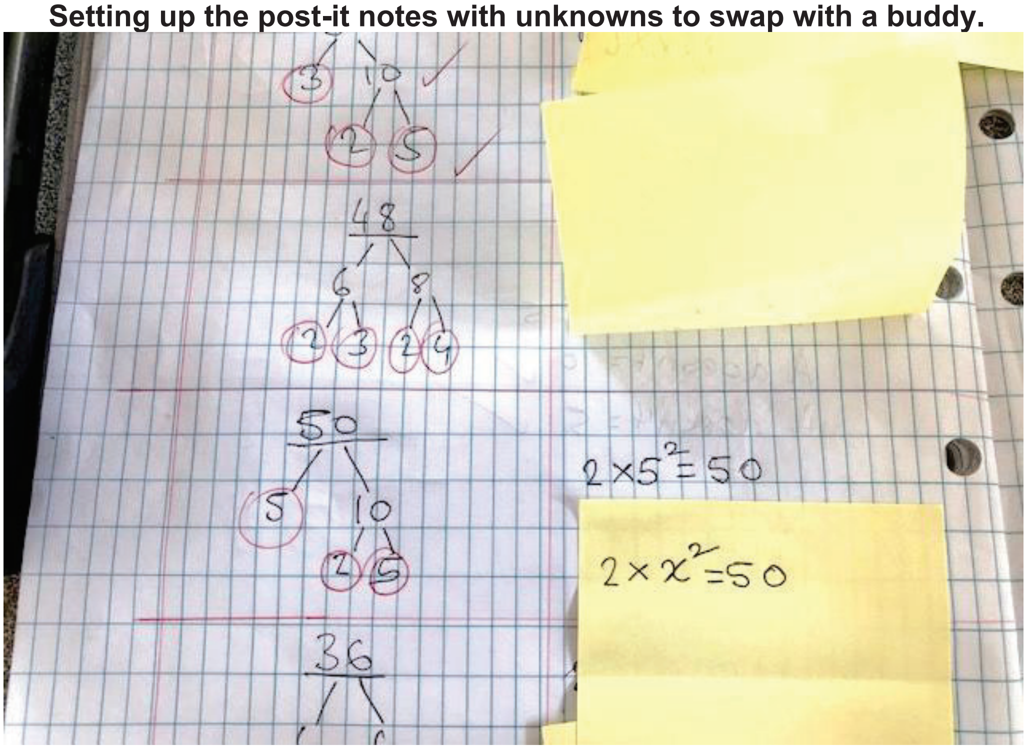
The truth is, of course, this is not actually possible. Try to tell them this fib convincingly and challenge extension students to realise and justify why it cannot be so (because if you multiplied the factors together, they could only ever make one total, due to the **commutative law**). The arithmetic laws (like commutativity) hold true for all numbers – so never get tricked by a sneaky teacher who says there is an example where it doesn't work (unless it involves '0' or '1' where often the laws do not apply, like with prime numbers).

Extension 2 – Year 8/9 level algebra: Publish the prime factorisation for your gallery on a post-it note. However, make one of the factors an algebraic unknown, such as 'm' for 'mystery number,' or the first letter of your name.

Swap post-it notes with an extension-level partner, who then can use substitution, trial and error and simplifying of the equation to try to solve the swapped post-it note. Below 'x' is replacing '3' in the prime factorisation:

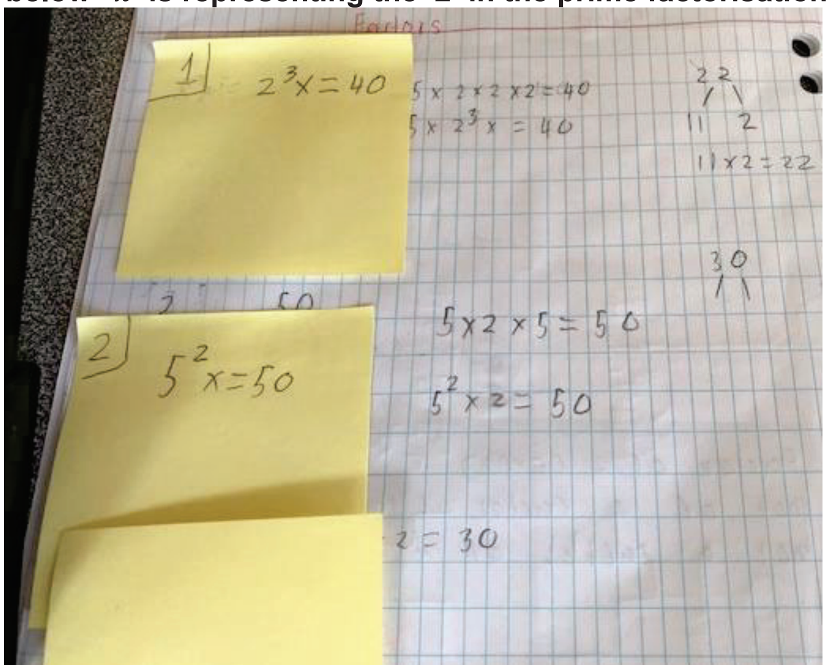


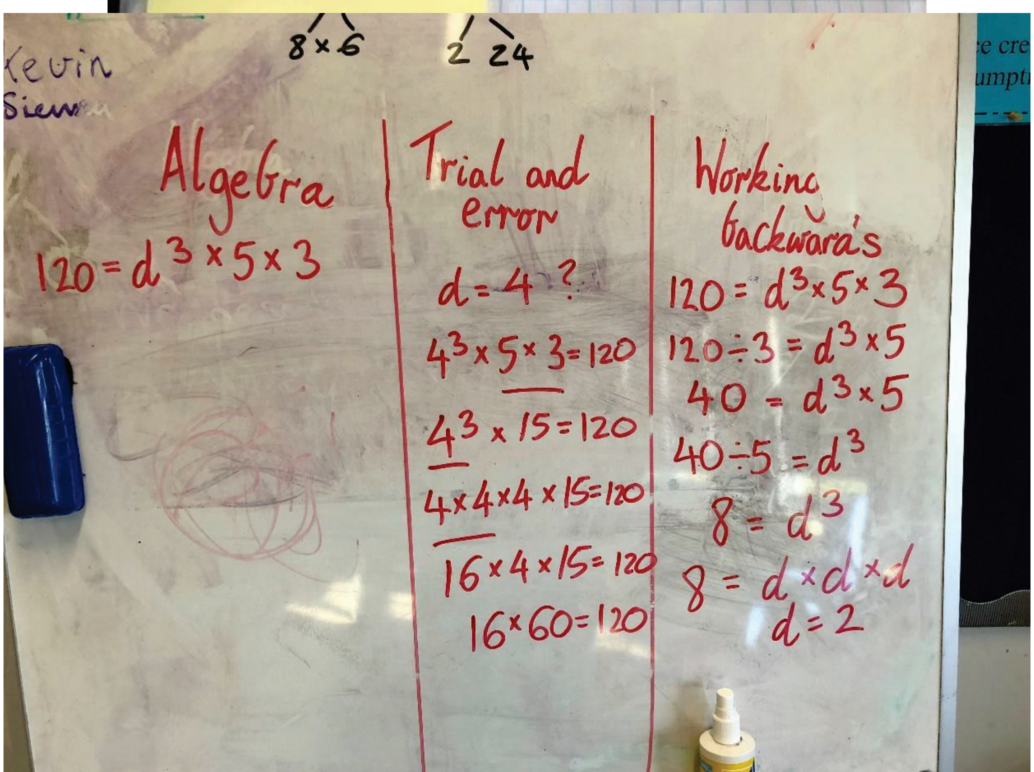
Setting up the post-it notes with unknowns to swap with a buddy.

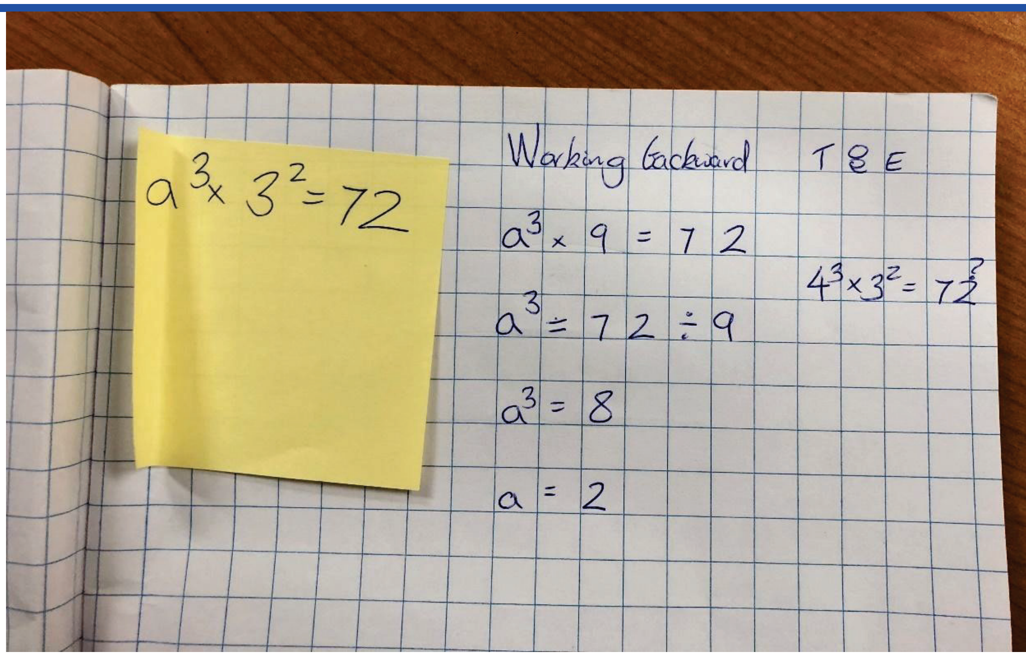


Above, 'x' is replacing '5' in the prime factorisation of 50 for the extension partner who receives this post-it note to solve.

Below, ‘x’ is replacing ‘5’ in the prime factorisation of 40, and then further below ‘ x’ is representing the ‘2’ in the prime factorisation of 50.





	 **Working backwards or using trial and error to solve the unknown**
	Extension 3: Can negative integers be prime? Give students 5-10 minutes of brainstorm time, including considering multiplying a negative by a positive makes a negative, so the equations that total to -3 include -1 x 3, -3 x 1, so the factors would be -3, -1, 1, 3. Primes are defined as having two factors only (1 and itself), so for this reason, most mathematicians conclude that it is impossible to consider negative integers as prime (unless you think in absolute values for the factors, which is discussed below). Read some mathematical discourse and debate on the subject here: https://www.go4expert.com/forums/negative-prime-t18798/

