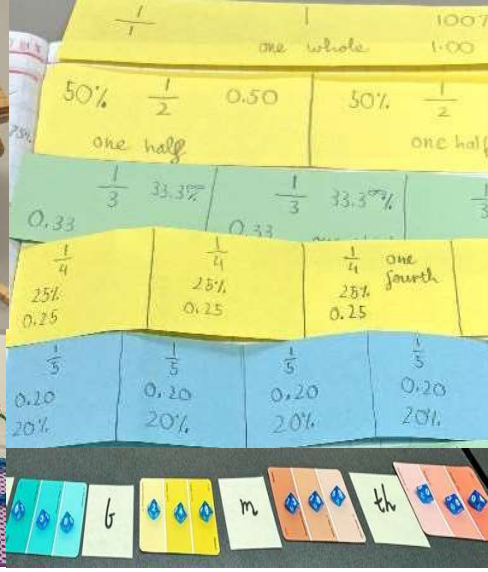


Real-Life Numeracy Years 3-6

Planning Package

Sequential units with hands-on, real-life numeracy for Year 3, Year 4, Year 5 and Year 6 students

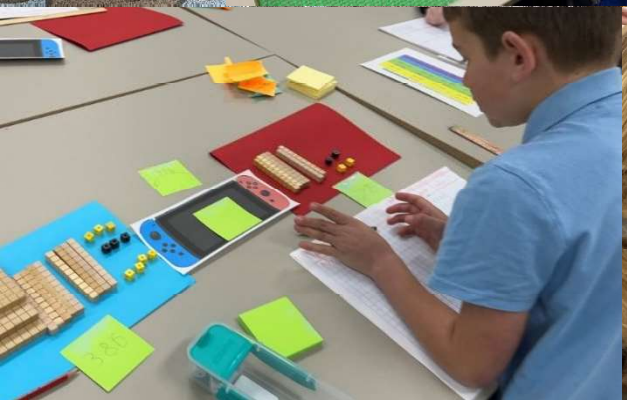
Ten years of development in Australian classrooms.

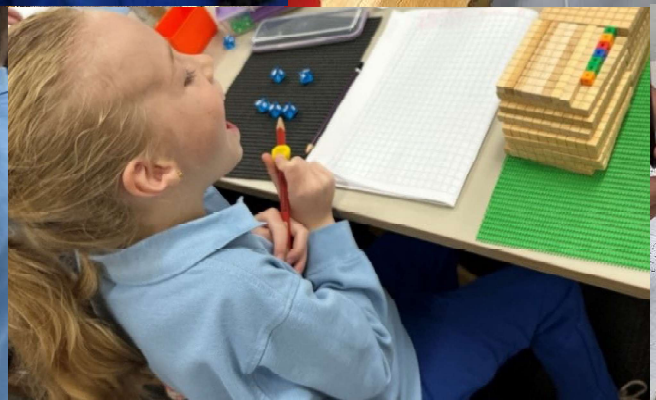
Genuinely high engagement and conceptual understanding in middle to upper primary numeracy.

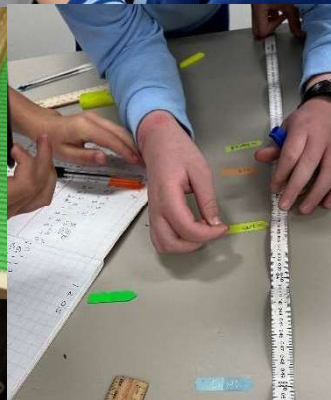
Comprehensive differentiation for wide ranges: Pre-planned and workable enabling and extending prompts for every lesson.

High-impact, high-relevance professional learning on a daily basis to support planning.

Comprehensive diagnostic and formative assessments to target each sequential point-of-need.







Please note: It is not intended for teachers to attempt to deliver every lesson in this sequence, nor read the unit in full.

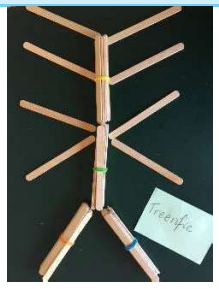
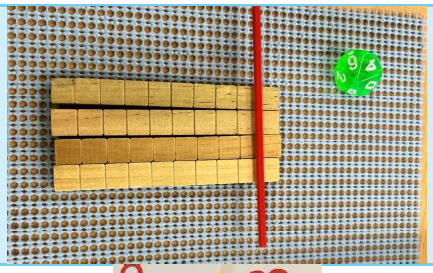
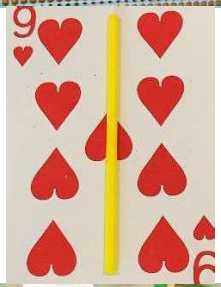
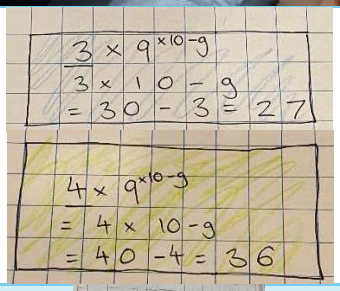
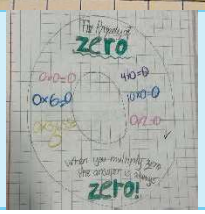
Units are designed as **a menu of options**, depending on the points-of-need for each class, with enabling and extending prompts included for every lesson.

Please choose lesson options based on assessed points-of-need (units are directly linked to the assessments), using either Top Ten's or other **strategy-focused diagnostic pre-assessments.** We recommend avoiding multiple-choice/click-the-answer tests, as numeracy as a discipline grows students' reasoning and thinking skills, ability to explain and show strategies, as well as deep conceptual understanding. Answers alone are not the ultimate goal, or a worthy aspiration in the absence of student reasoning.

Please also select lessons that best suit students' interests and your own creativity and passion. Units are designed to share the wisdom of practice, while respecting and safeguarding the professional role of the teacher as the ultimate best judge of students' needs.

Adjust how many lessons you deliver based on student progress throughout the unit, which can be tracked using the formative assessment folder.

Multiplication Unit 3 for Year 3 and Year 4 Tens Strategy Family Hyperlinked Table of Contents

Curriculum Links for Year 3 and Year 4 Pages 4-8	**Formative Assessment** Page 9 **Strategy Flipbook Journals** Pages 11-12 **Teaching Tips** Pages 13-17
Warm-up Games: Break the Array, Number Talks, Place Value Patterns, Spinners Pages 18-34	
Lesson Sequence **Underlined lessons are highly recommended**	
Lesson 1 Tens Apple Trees Pages 35-37	
Lesson 2 Tens Strategy Pages 38-41	
Lesson 3 Nines Strategy Pages 42-50	
Lesson 4 Fives Strategy V1 Pages 51-55	
Lesson 5 Fives Strategy V2 Pages 56-62	
Lesson 6 Land Grab Pages 63-67	
Lesson 7 Strategy Flipbooks Pages 68-74	
Lesson 8 Multiplying by 0 and 1 Pages 75-78	

Multiplication Unit for Year 3 and 4

Curriculum Links for the following lessons

This unit is recommended for Year 3 and Year 4 students.

Australian Curriculum V9 AC9M3A03 and Victorian Curriculum Version 2.0 (VC2M3A03)
Algebra – Level 3: Recall and demonstrate proficiency with multiplication facts for 3, 4, 5 and 10; extend and apply facts to develop the related division facts

- using concrete or virtual materials, groups and repeated addition to recognise patterns and establish the 3, 4, **5 and 10** multiplication facts; for example, using the language of '3 groups of 2 equals 6' to develop into '3 twos are 6' and extend to establish the 3 × 10 multiplication facts and related division facts
- **recognising that when they multiply a number by 5, the resulting number will either end in a 5 or a zero; and using a calculator or spreadsheet to generate a list of the multiples of 5 to develop the multiplication and related division facts for fives**
- practising calculating and deriving multiplication facts for 3, 4, **5 and 10,** explaining and recalling the patterns in them and using them to derive related division facts
- systematically exploring algorithms used for repeated addition, comparing and describing what is happening, and using them to establish the multiplication facts for 3, 4, **5 and 10;** for example, following the sequence of steps, the decisions being made and the resulting solution, recognising and generalising any emerging patterns

Australian Curriculum V9 AC9M4N06 and Victorian Curriculum Version 2.0 (VC2M4N06)
Number – Level 4: Develop efficient mental and written strategies and use appropriate digital tools for solving problems involving addition and subtraction, and multiplication and division where there is no remainder

- **using physical or virtual materials to demonstrate doubling and halving strategies for solving multiplication problems; for example, for 5 × 18, using the fact that double 5 is 10 and half of 18 is 9; or using 10 × 18 = 180, then halving 180 to get 90; or applying the associative property of multiplication, where 5 × 18 becomes 5 × 2 × 9, then 5 × 2 × 9 = 10 × 9 = 90 so that 5 × 18 = 90**
- using an array to represent a multiplication problem, connecting the idea of how many groups and how many in each group with the rows and columns of the array, and writing an associated number sentence
- using materials or a diagram to solve a multiplication or division problem, by writing a number sentence and explaining what each of the numbers within the number sentence refers to
- representing a multiplicative situation using materials, array diagrams and/or a bar model, and writing multiplication and/or division number sentences, based on whether

the number of groups, the number per group or the total is missing, and explaining how each number in their number sentence is connected to the situation

- using place value partitioning, basic facts and an area or region model to represent and solve multiplication problems; for example, for 16 × 4, thinking 10 × 4 and 6 × 4, then 40 + 24 = 64, or a double double strategy where double 16 is 32, double this is 64, so 16 × 4 is 64

Australian Curriculum V9 AC9M4A02 and Victorian Curriculum Version 2.0 (VC2M4A02)
Algebra – Level 4: Recall and demonstrate proficiency with multiplication facts up to 10 × 10 and related division facts, and explain the patterns in these; extend and apply facts to develop efficient mental and written strategies for computation with larger numbers without a calculator

- using arrays on grid paper or created with blocks or counters to develop, represent and explain patterns in multiplication facts up to 10 × 10; and using the arrays to explain the related division facts
- using materials or diagrams to develop and record multiplication strategies such as doubling, halving, commutativity and adding one more or subtracting from a group to reach a known fact; for example, creating multiples of 3 on grid paper and doubling to find multiples of 6, and recording and explaining the connections to the × 3 and × 6 multiplication facts: 3, 6, 9, … doubled is 6, 12, 18, …
- using known multiplication facts for 2, 3, **5 and 10 to establish multiplication facts** for 4, 6, 7, 8 and **9** in different ways; for example, **using multiples of 10 to establish the multiples of 9 as 'to multiply a number by 9 you multiply by 10 then take the number away': 9 × 4 = 10 × 4 − 4, so 9 × 4 is 40 − 4 = 36;** or using multiples of 3 as 'to multiply a number by 9 you multiply by 3, and then multiply the result by 3 again'
- using arrays and known multiplication facts for twos and fives to develop the multiplication facts for sevens, applying the distributive property of multiplication; for example, when finding 6 × 7, knowing that 7 is made up of 2 and 5, and using an array to show that 6 × 7 is the same as 6 × 2 + 6 × 5 = 12 + 30, which is 42

Additional elaborations in the Australian Curriculum Version 9:

- using known multiplication facts up to 10x10 and the inverse relationship of multiplication and division to establish corresponding division facts
- designing, creating and playing instructive card games that involve the recall, recognition and explanation of the 10x10 multiplication facts and related division facts

Australian Curriculum V9 AC9M4N08 and Victorian Curriculum Version 2.0 (VC2M4N09)
Number – Level 4: Use mathematical modelling to solve practical problems that involve additive and multiplicative situations, including financial contexts; formulate the problems using number sentences and choose efficient calculation strategies, using digital tools where appropriate; interpret and communicate solutions in terms of the situation

- modelling and solving multiplication problems involving money, such as buying 5 toy scooters for $96 each, using efficient mental strategies and written jottings to keep track if needed; for example, rounding $96 up to $100 and subtracting 5 × $4 = $20, so 5 × $96 is the same as 5 × $100 less $20, giving the answer $500 − $20 = $480

- modelling situations by formulating comparison problems using number sentences, comparison models and arrays; for example, 'Ariana read 16 books for the "readathon"; Maryam read 4 times as many books. How many books did Maryam read?' using the expression 4 × 16 and using place value partitioning, basic facts and an array, thinking 4 × 10 = 40 and 4 × 6 = 24, so 4 × 16 can be written as 40 + 24 = 64

Australian Curriculum V9 AC9M5N06 and Victorian Curriculum Version 2.0 (VC2M5N06) Number – Level 5: Solve problems involving multiplication of larger numbers by one- or two-digit numbers, choosing efficient mental and written calculation strategies and using digital tools where appropriate; check the reasonableness of answers

- solving multiplication problems like 15 × 16 by thinking of factors of both numbers, 15 = 3 × 5, 16 = 2 × 8, and rearranging the factors to make the calculation easier, 5 × 2 = 10, 3 × 8 = 24 and 10 × 24 = 240
- using an array to show place value partitioning to solve multiplication, such as 324 × 8, thinking 300 × 8 = 2400, 20 × 8 = 160, 4 × 8 = 32 then adding the parts, 2400 + 160 + 32 = 2592; and connecting the parts of the array to a standard written algorithm
- using different strategies used to multiply numbers, and explaining how they work and if they have any limitations; for example, discussing how the Japanese visual method for multiplication is not effective for multiplying larger numbers

Western Australian Curriculum Number and Algebra – Level 3: Recall multiplication facts of two, three, five and ten and related division facts (ACMNA056)

- establishing multiplication facts using number sequences

Western Australian Curriculum Number and Algebra – Level 4: Recall multiplication facts up to 10 × 10 and related division facts (ACMNA075)

- using known multiplication facts to calculate related division facts

Western Australian Curriculum Number and Algebra – Level 4: Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and for division where there is no remainder (ACMNA076)

- using known facts and strategies, such as commutativity, **doubling and halving for multiplication,** and connecting division to multiplication when there is no remainder

NSW Maths Syllabus Whole Numbers – Stage 2 – Multiplicative relations A

Generate and describe patterns

- Model, describe and record patterns of multiples
- Create and continue a variety of number patterns that increase or decrease by a constant amount
- Recognise the significance of the final digit of a whole number in determining whether a given number is even or odd (Reasons about relations)
- Recognise the connection between even numbers and the multiplication facts for 2 (Reasons about relations)
- **Investigate the result of multiplying by one and zero (Reasons about relations)**

Use arrays to establish multiplication facts from multiples of 2 and 4, 5 and 10

- Create and represent multiplicative structure, using the term multiples when connecting grouping to arrays
- Use the array structure to coordinate the number of groups with the number in each group
- Record the first 10 multiples formed by counting by twos, fours, **fives and tens**
- Relate *doubling* to multiplication facts for multiples of 2
- Recognise that doubling is multiplying by 2 and *halving* is dividing by 2 (Reasons about relations)
- Recognise the relationship between one multiple and its double (Reasons about relations)
- Model square numbers and record in numerical and diagrammatic form

Recall multiplication facts of 2 and 4, **5 and 10** and related division facts

- Recognise and use the symbols for multiplied by x, divided by ÷ and equals =
- Link multiplication and division fact families using arrays
- Generate multiplication fact families for multiples of 2 and 4, **5 and 10**
- Model and apply the commutative property of multiplication

Represent and solve problems involving multiplication fact families

- Describe multiplication problems using *for each* and *times as many*
- Find the total of partially covered arrays
- Apply the inverse relationship of multiplication and division (Reasons about relations)

NSW Maths Syllabus Whole Numbers – Stage 2 – Multiplicative relations B

Investigate number sequences involving related multiples

- Generate number patterns using related multiples
- **Investigate number patterns involving related multiples**

Use known number facts and strategies

- Apply the known strategy of doubling to connect multiples of 3 to 6 and 4 to 8 (Reasons about relations)
- **Use known facts to find unknown multiples (Reasons about relations)**

Use the structure of the area model to represent multiplication and division

- Create and represent multiplicative structure, moving from arrays to partially covered area models

Use number properties to find related multiplication facts

- Use the commutative property of multiplication
- **Use the associative property within multiplication to regroup the factors (Reasons about structure)**
- **Use flexible partitioning within multiplication (Reasons about relations)**
- Generate and recall multiplication fact families up to 10 x 10

Operate with multiples of 10

- **Use multiplication facts with multiples of 10 to multiply a one-digit number by a multiple of 10**
- **Use place value to rename groups of 10 to multiply**
- Apply the commutative and associative properties to multiply by multiples of 10

Represent and solve word problems with number sentences involving multiplication or division

- Use the equals sign to record equivalent number relationships involving multiplication (Reasons about relations)
- Complete number sentences involving multiplication and division by calculating missing numbers (Reasons about relations)
- Represent and solve multiplication and division (both sharing and grouping) word problems using number sentences

Formative Assessment

A formative assessment cross-check is available in this unit's folder with progressive learning goals and specific success criteria for this unit. This includes a concise teacher assessment record sheet focused on students learning strategy families.

Multiplicative Strategies

Students	x2 double it DOUBLING (d)	X4 double double DOUBLING (dd)	X8 double double double DOUBLING (ddd)	X10 place value TENS FAMILY (pv)	X9 x10 – group TENS FAMILY (pv – g)	X5 pv ½ it (x10, halve it) TENS FAMILY (pv ½)	X3 double + group DOUBLING (d + g)	X6 multiple strategies (x3d or x5+g or similar)	X7 memorise 7x7, all others turnarounds or x5 + c or similar

There is also a strategy and fluency student goal tracker:

Multiplicative strategies

Strategy families I can solve fast:

-
-
-
-

I am currently working on:

-
-

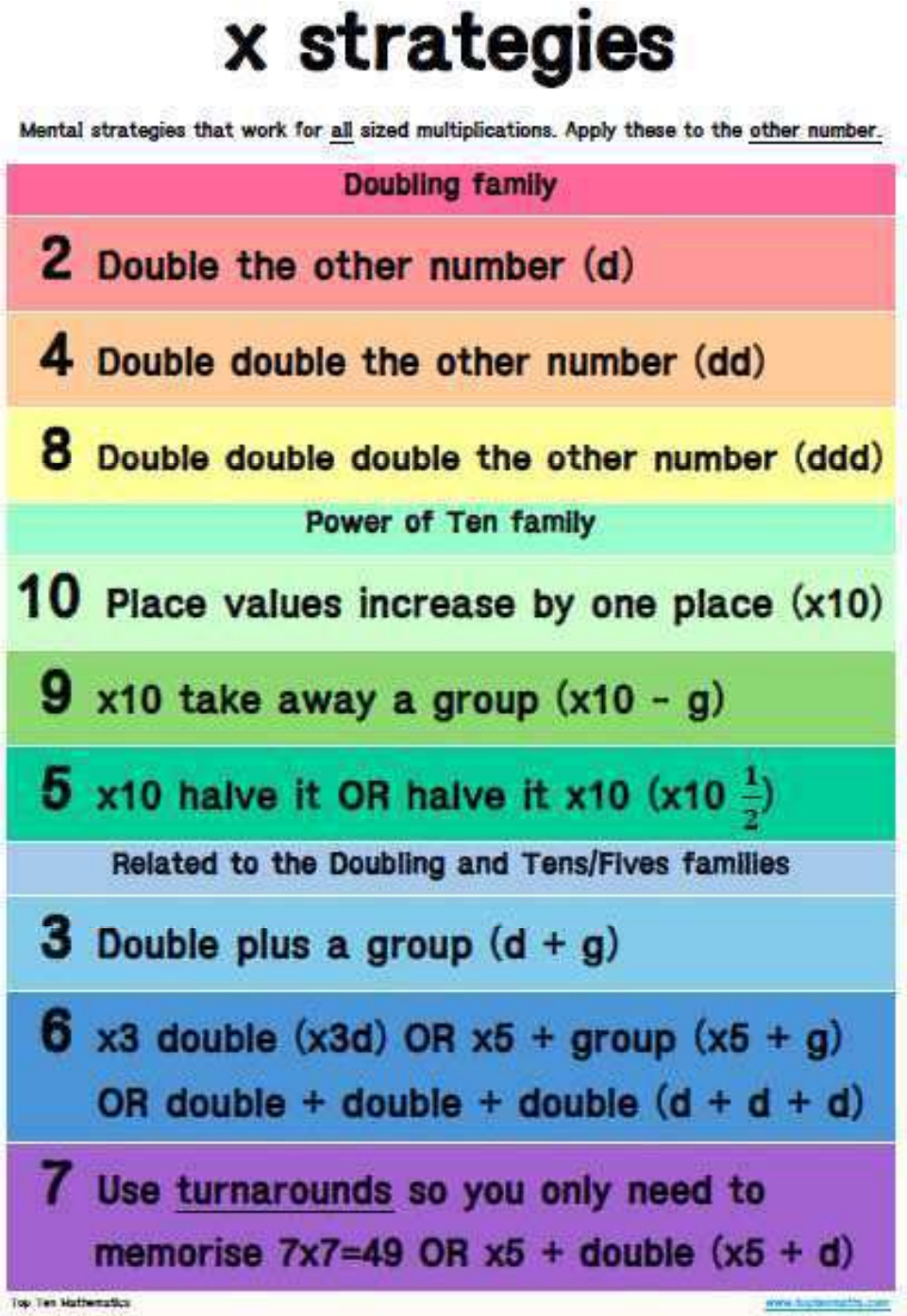
Name:

Shade which ones you know the strategies for and can do fast:

	0	1	2	3	4	5	6	7	8	9	10
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											

The trickiest facts I want to memorise next:

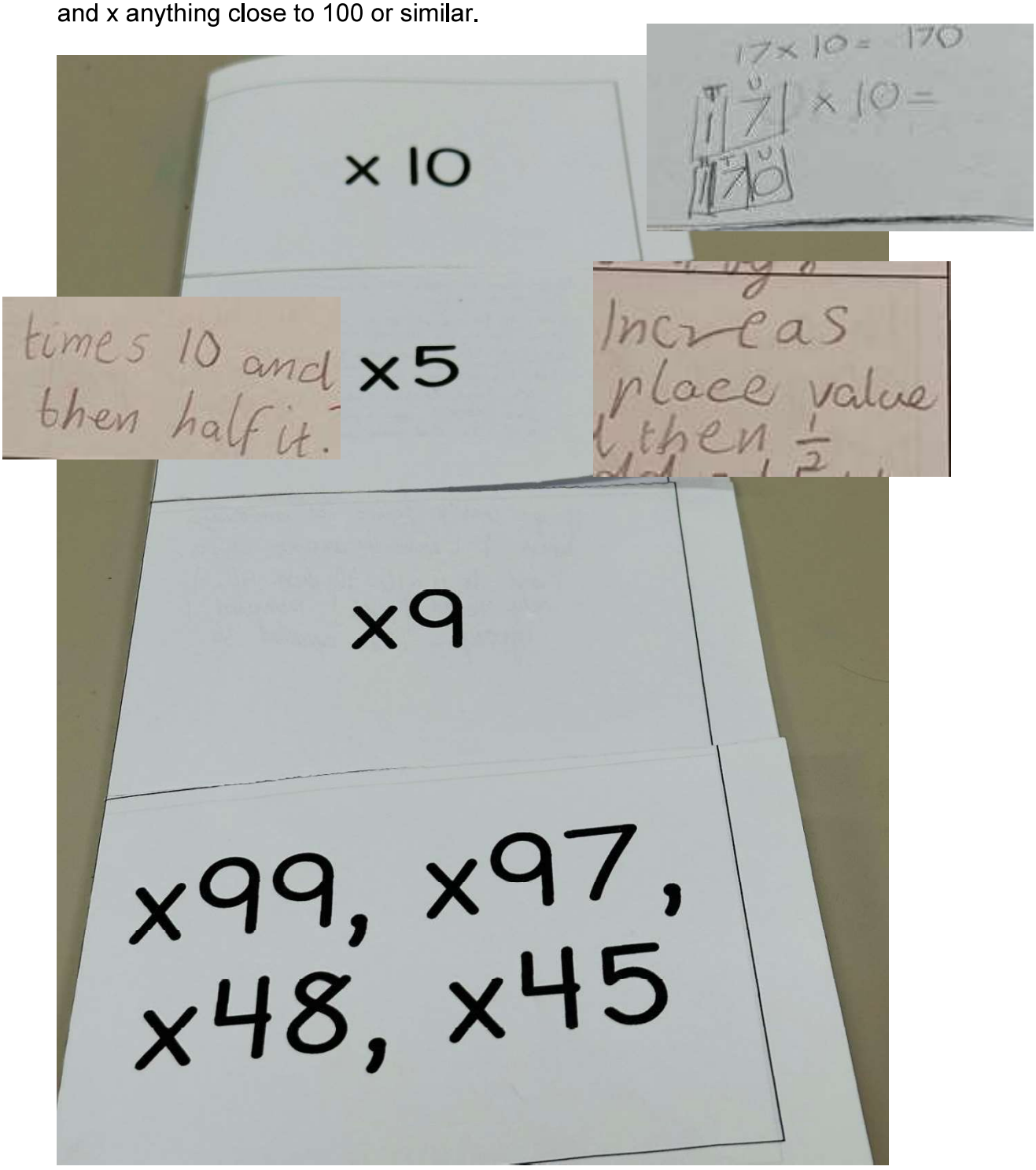
Multiplicative Strategies Anchor for Students



Template available

Student Strategy Flipbooks to record what they notice throughout array-based practice

Flipbook strategy journal, which guides students through the progression of x10, x9, x5, and x anything close to 100 or similar.



Tens Family Year 3/4 Lesson 3

Nines Strategy

Learning intention: Understand the x10 take away a group strategy for the nines.

Maths vocabulary: x10 – g, strategy for the nines, array, take away

Brainstorm things that come in nines:

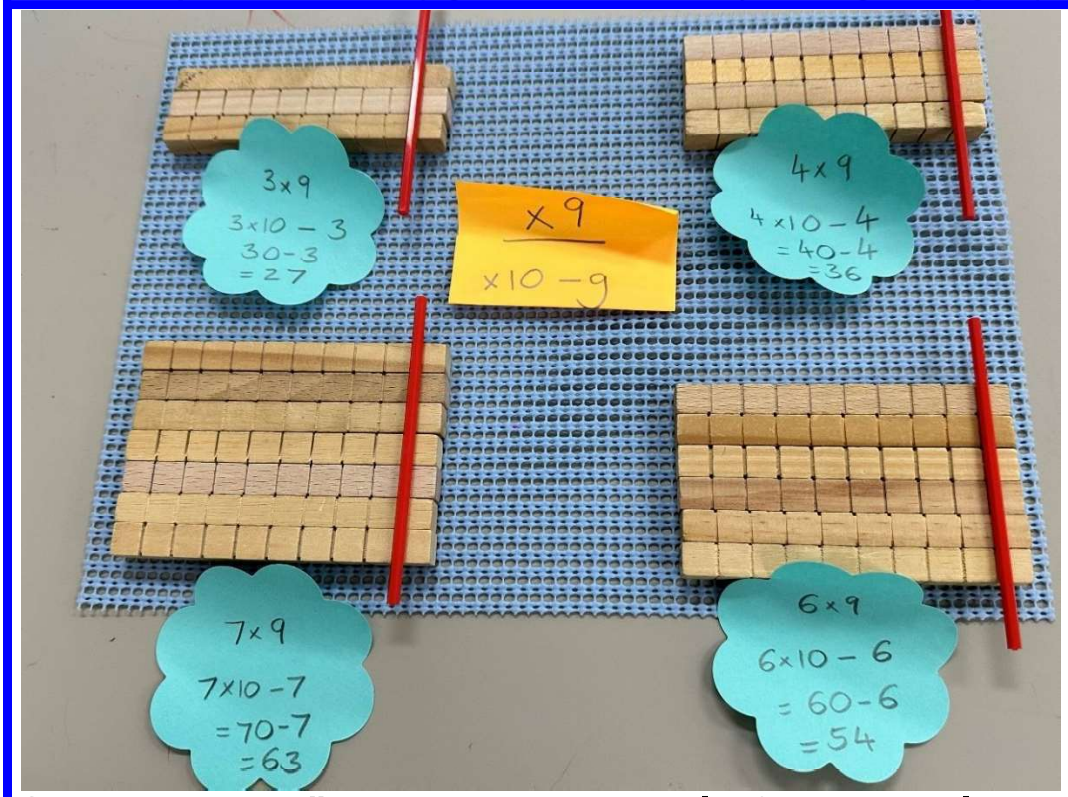
- nine squares in naughts and crosses
- lives of cats
- nine members in The Fellowship of the Ring
- Santa has nine reindeer
- nine innings in baseball
- nine Greek muses
- nine family members in the Brady Bunch

Lesson summary: Students practise the x10 – group strategy for the nines, using arrays as tens place value blocks and counters, visually 'slicing' the extra ones off each row, so that the tens become nines.

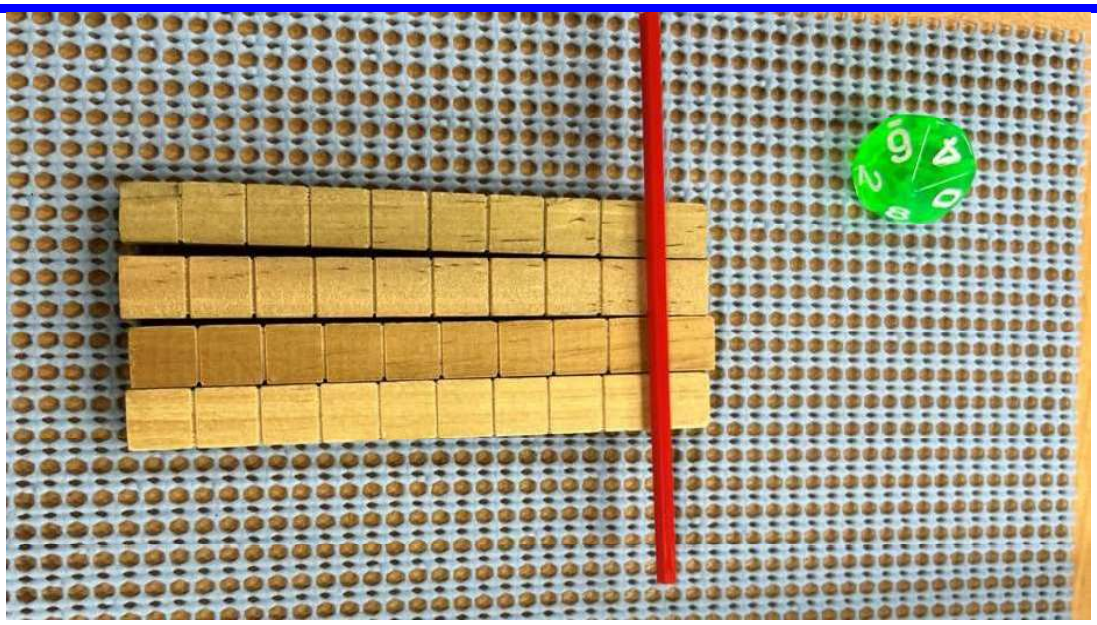
Materials:

- Tens place value blocks (**wooden MAB** – wooden is critical, as the notches are much clearer than plastic MAB versions).
- Red sticks.
- Grip mats.
- Two-sided counters.
- Post-it notes.

Best set-up: Students set up galleries and solve by thinking x10 – group.



Students create galleries using tens, then solve for nines by slicing off one from each row of ten. This shows how many need to be taken away to reduce the answer to the multiplication by ten, to make the answer to the multiplication by nines: x10 then take away a group (x10 – g).



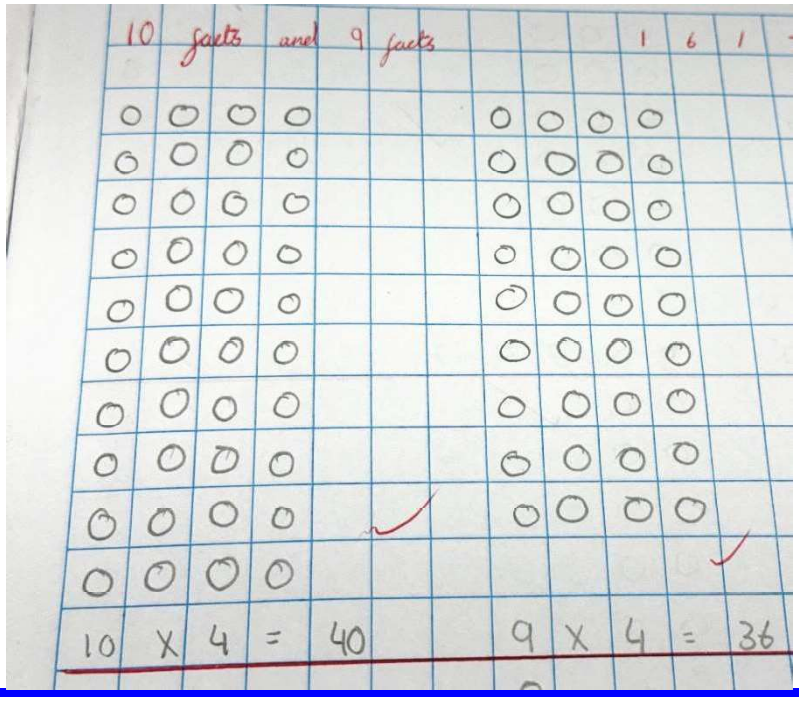
Roll the dice (rolled ‘4’).

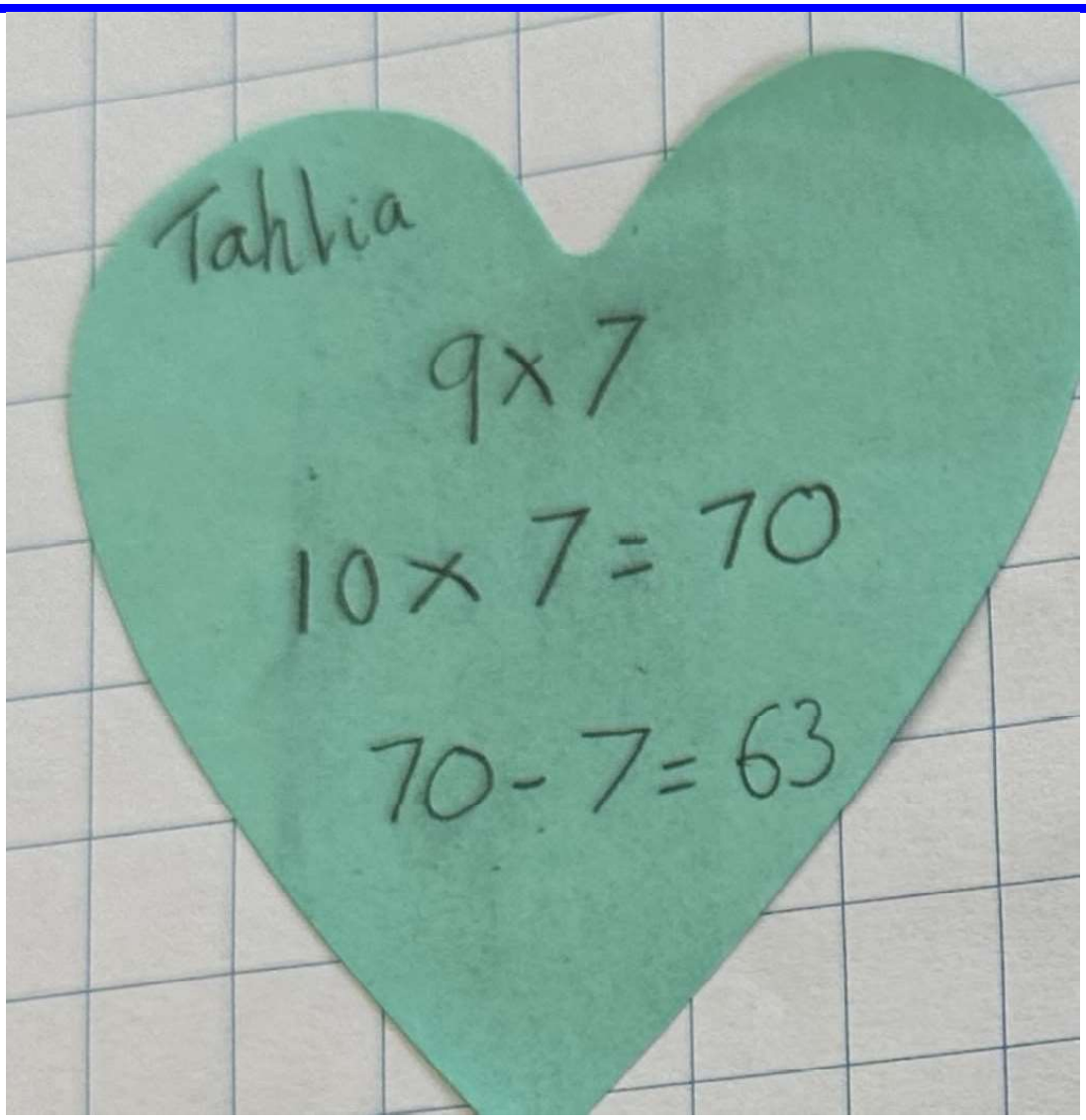
Pull out that number of tens to solve dice roll x 10 (4 x 10 = 40).

Then slice off a group to make it 4 groups of 9, instead of 4 groups of ten.

You can see why the answer is 40 – 4, so 36.

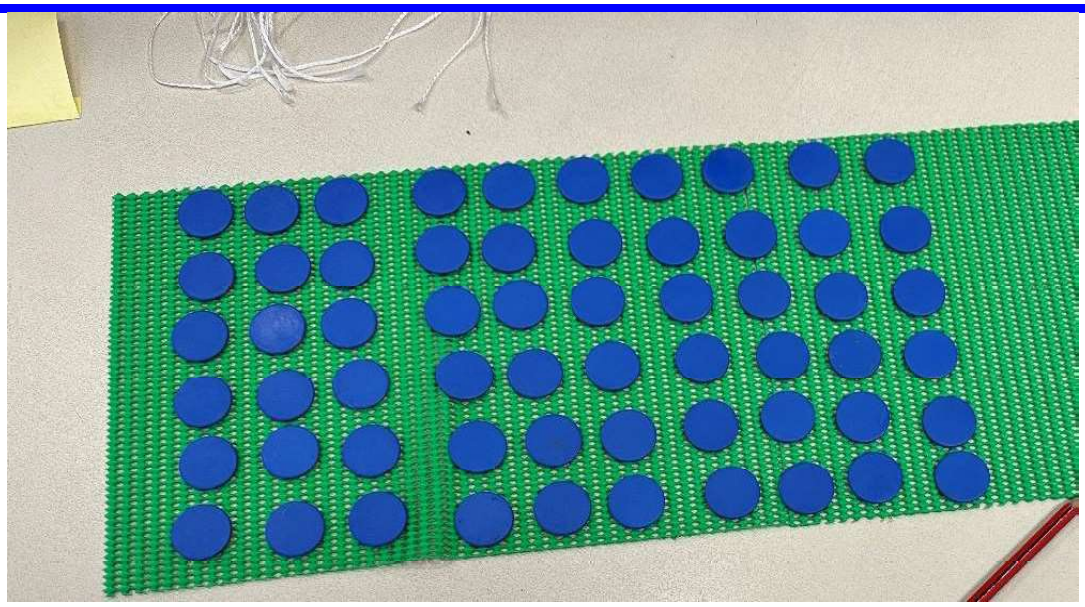
4 x 9 = 4 x 10 – 4
= 40 – 4
= 36





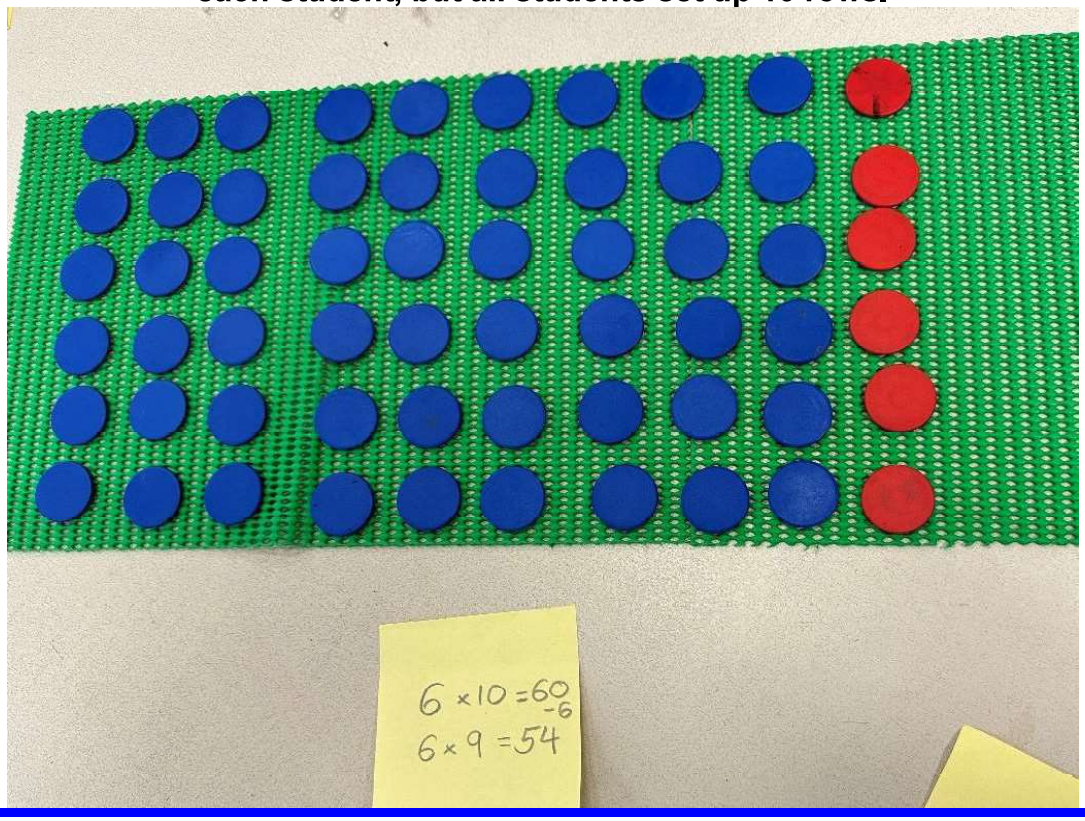
Think x10, solve for tens, then take away a group of the other number. For example, to solve 7 x 9, think 7 x 10, then take away the 7 = 63 (do not take away the 9 – take away the other number, because it is ten of that group, taking away 1 of each of that group, making it 9 of that group).

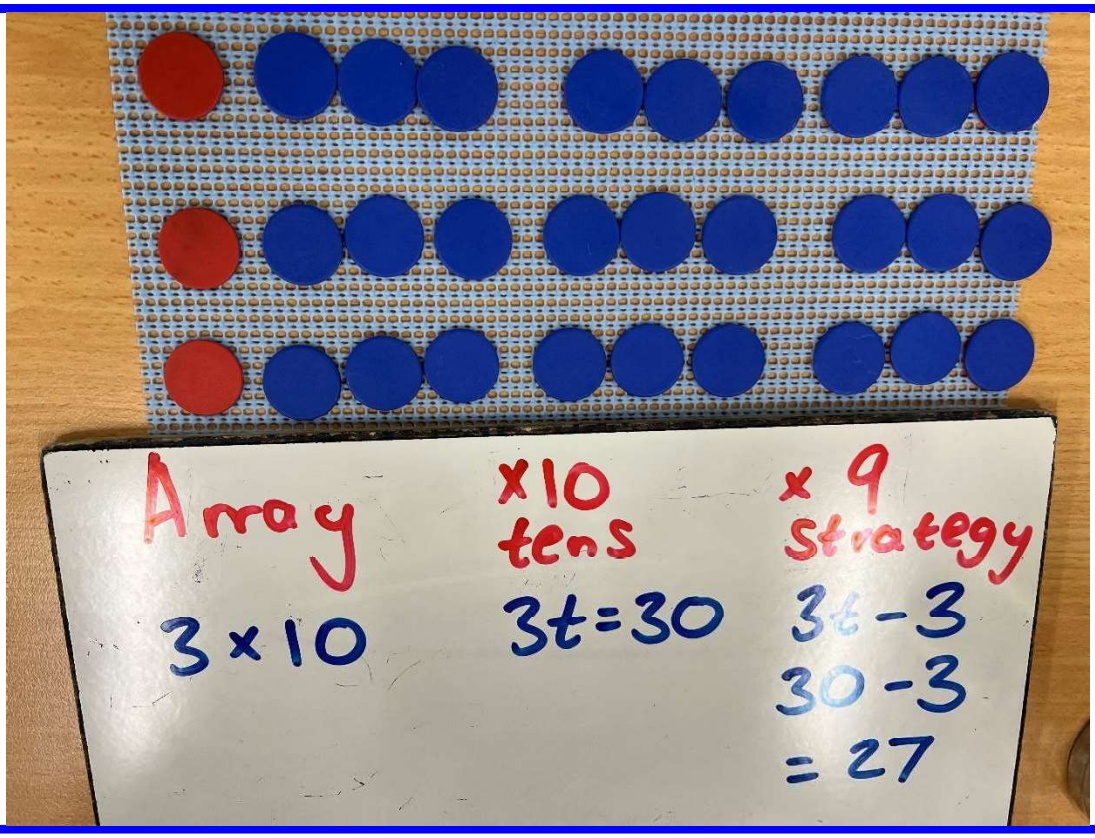
To check the answer, check that the digits add up to 9. For example, for 63, the 6 + 3 = 9, so you are almost certainly correct!



Make an array of 10 columns, then flip over one column to red to take it away, making it 9 rows to solve the answer to the nines.

If each student sets up a different array, there will be 30 arrays in the room for classmates to solve! Allocate a different number of rows to each student, but all students set up 10 rows.

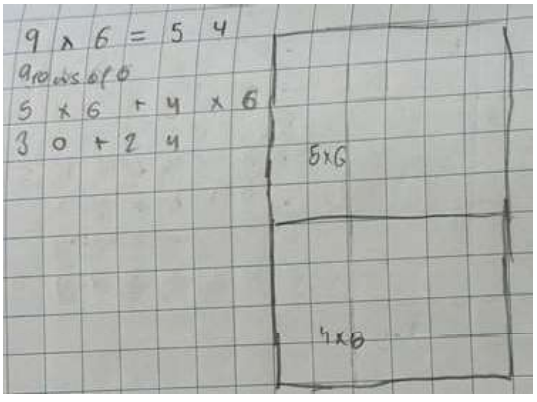


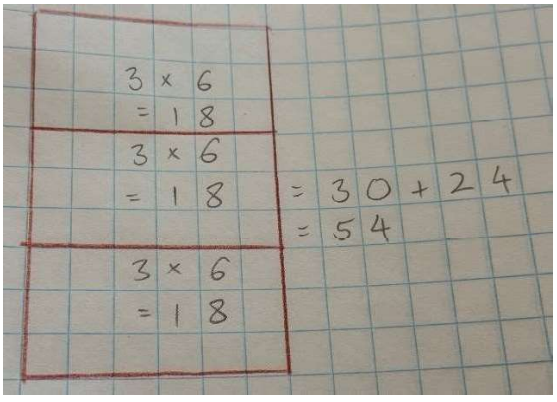


Support 1: Focus on consolidating and trusting the tens pattern first, otherwise the nines strategy will not be workable.

Support 2: If students are struggling, they may need to work on their 10 facts (playing games from the ***Early Years Pack – Addition Unit 5***), as they may not be able to fluently take away from 10 because they do not know their 10 facts, therefore cannot fluently take 7 away from 70.

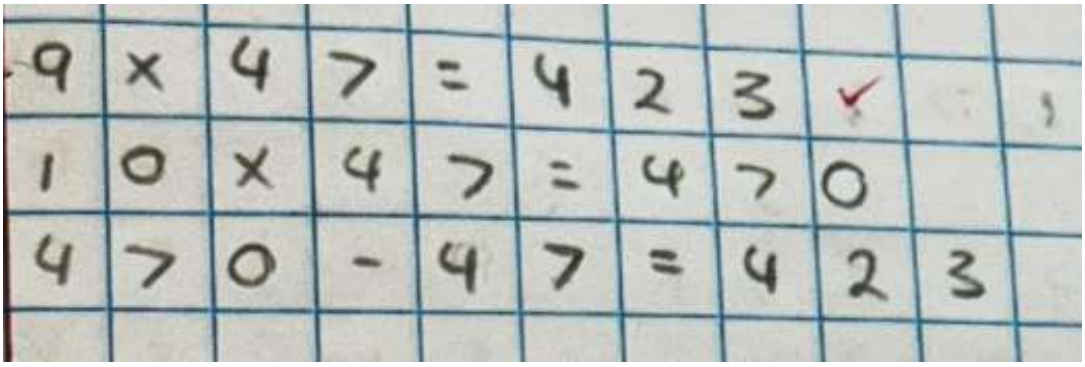
Extension 1: Solve it using at least two strategies, meaning students will need to come up with at least one other preferred strategy for x9:

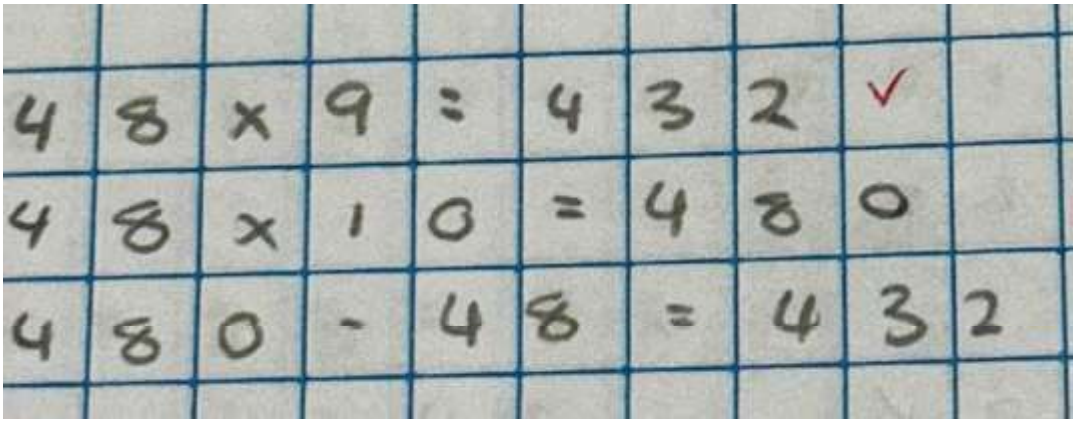




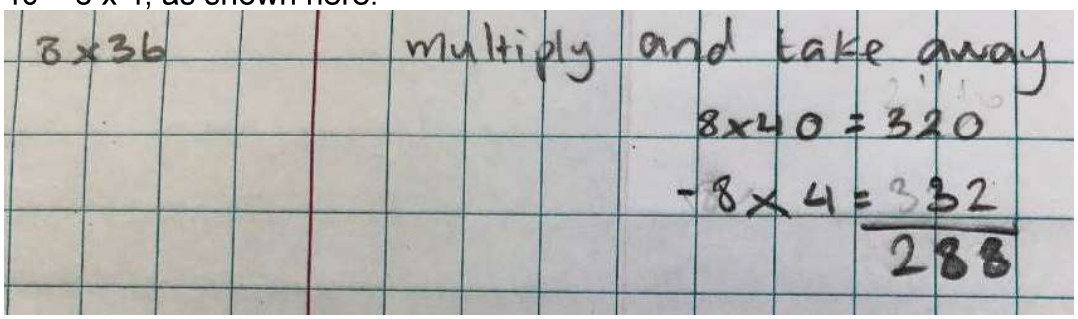
Examples: x5 + dd (x5 + x4), or x3 x3 (triple it, then triple it again), or similar, but do not tell students this, let them see how many diverse strategies they can brainstorm for the nines.

Extension 2: Use a similar compensation strategy for larger numbers.





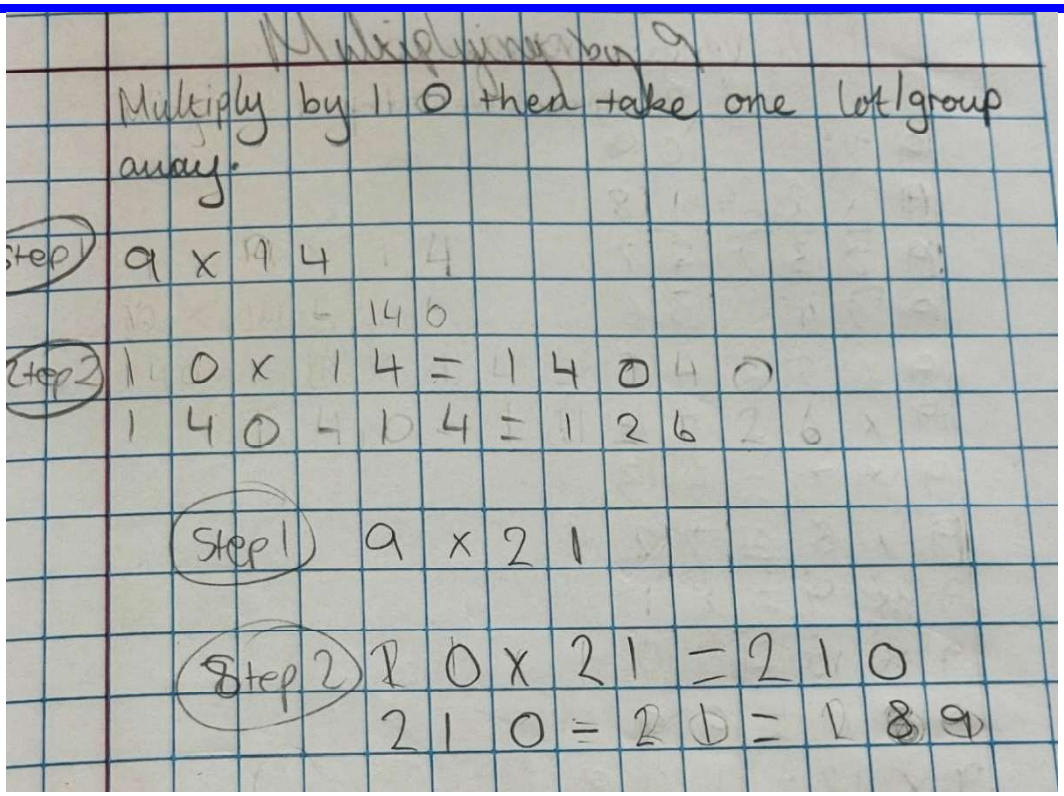
The nines strategy frontloads a compensation-based strategy for multiplications that are much larger. For example, to solve 8 x 36, solve 8 x 40 – 8 x 4, as shown here:

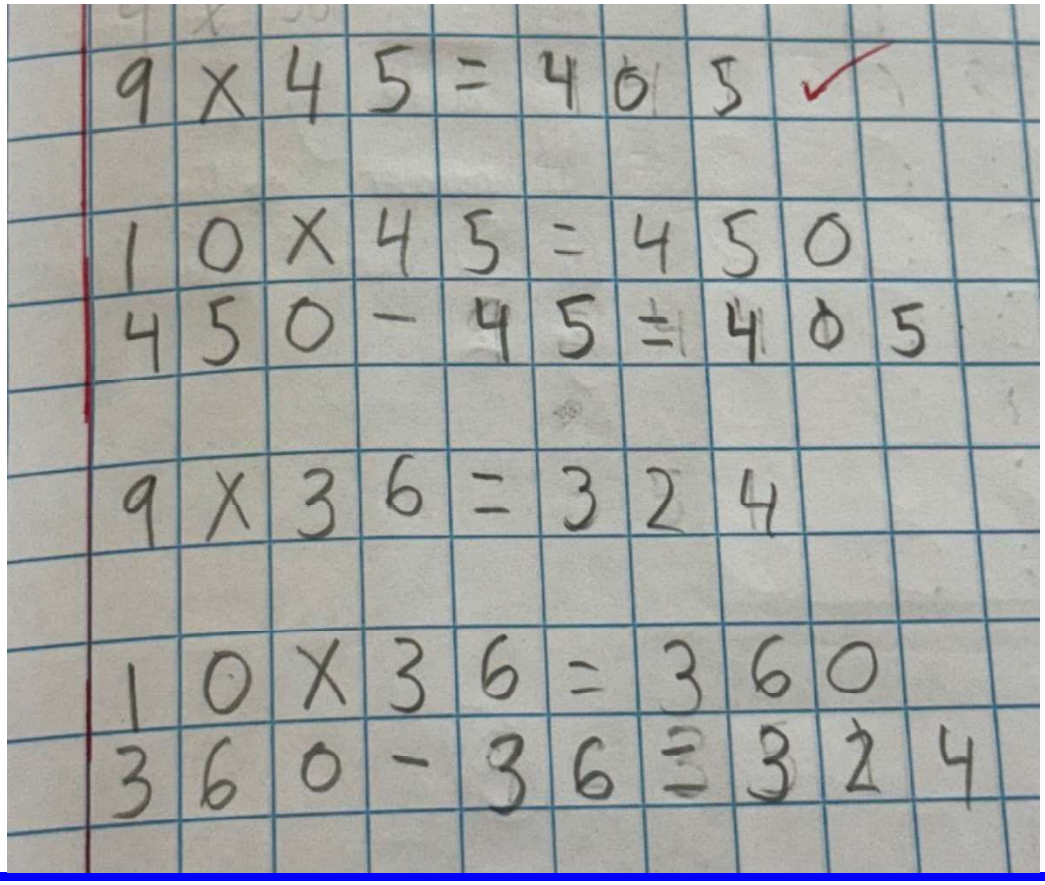


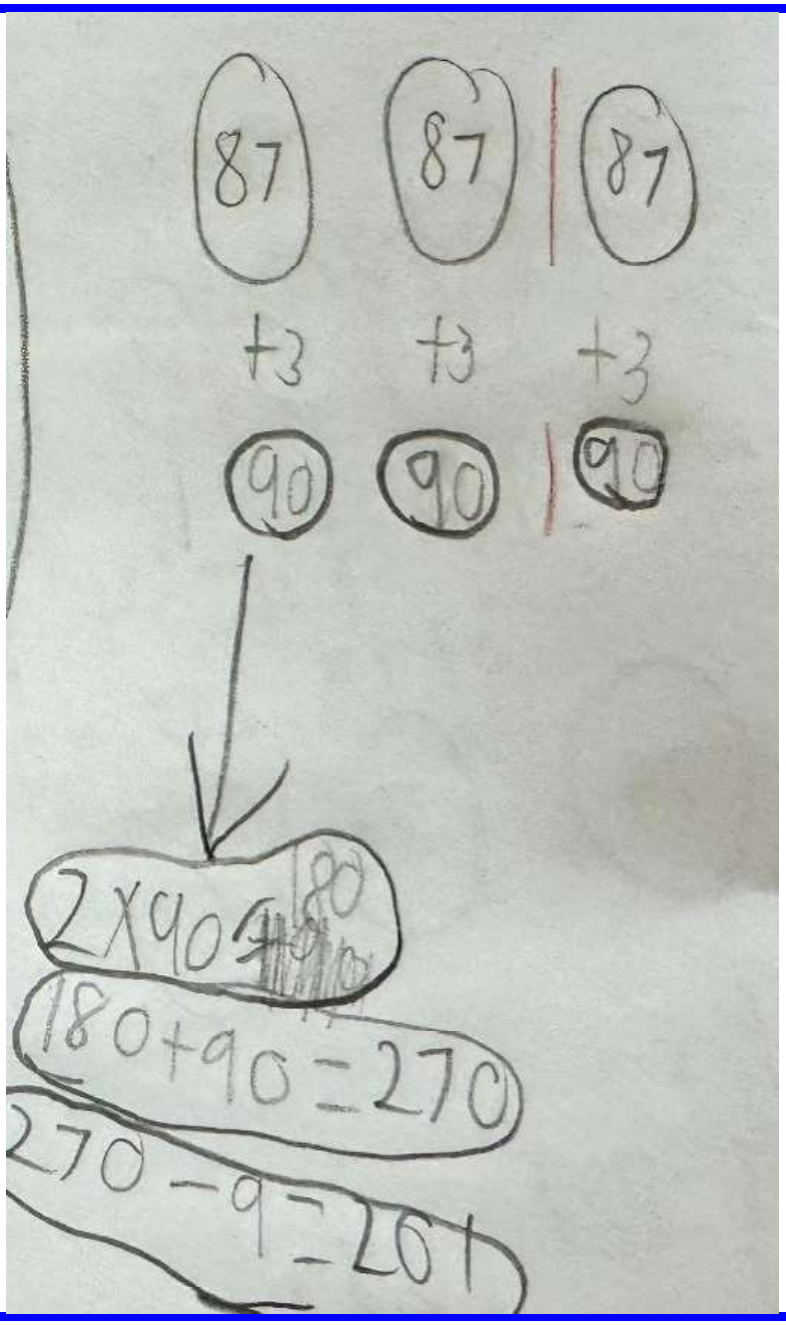
Create a list of problems for another like-ability extension buddy, where the numbers are 'ugly ducklings,' but could be made 'pretty swans' if they were rounded, then compensation was used, as in the work samples above.

Create a list of 5 problems, then swap with an extension buddy, to solve and check each answer. Then repeat with more challenging examples created for one another, like a game of endurance/survivor.

To link to real-life, also create worded problems for these equations.







A Year 2 extension student applying the nines strategy to solve 87 x 3 by solving 3 x 90, then taking away the 9 that was added on to round the 3 eighty sevens up to 90.

Tens family
Year 3/4
Lesson 4

Fives Strategy – Place value blocks split in half

Learning intention: Think x10 halve it to solve x5.

Maths vocabulary: halve, tens, place value, parts, split

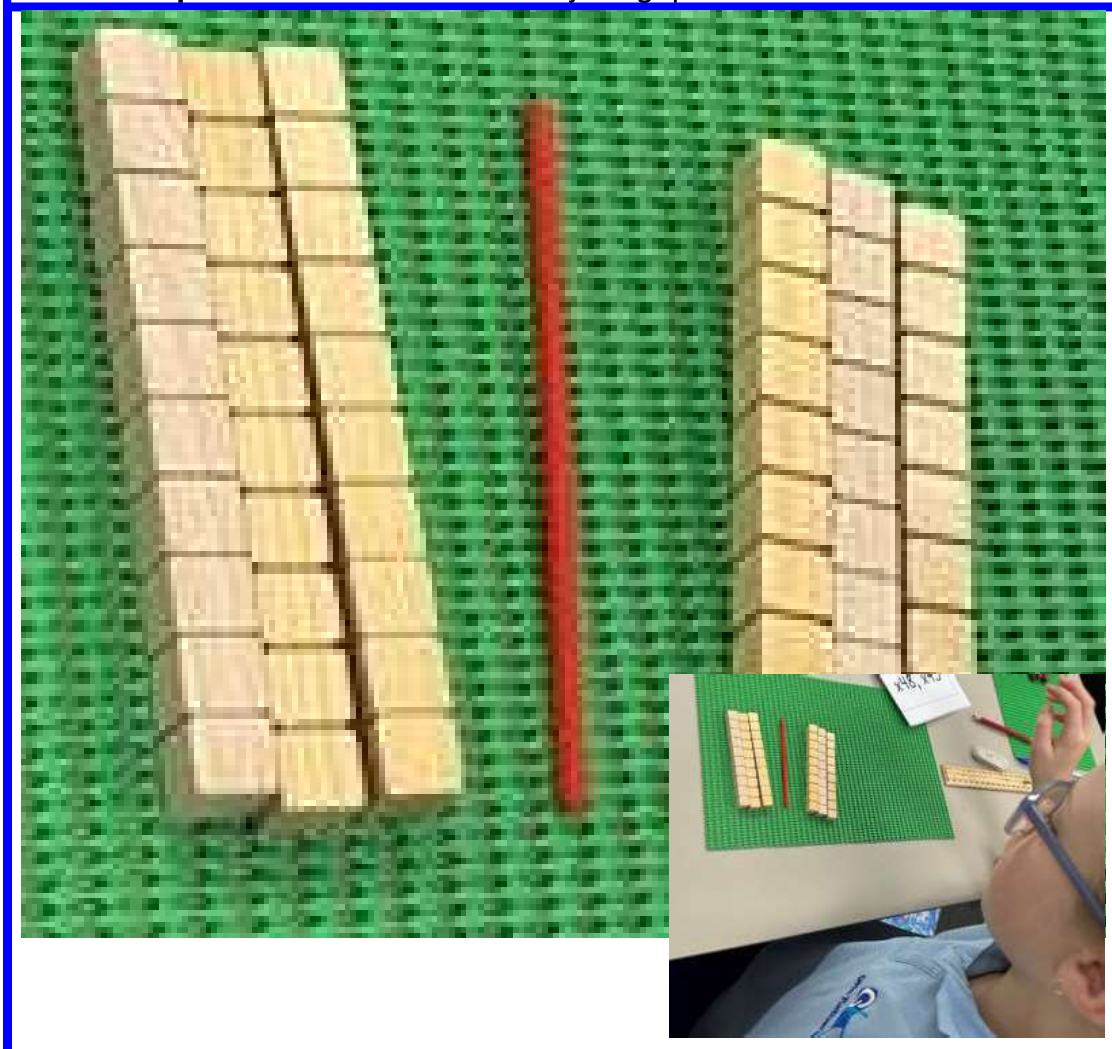
YouTube hook: To solve fives, split the tens in two or half, BUT these kids can't share: https://www.youtube.com/watch?v=KG73H5hvL6w

Lesson summary: Students split the answer to the tens in half to solve the fives, using an efficient strategy that can also work later for two-digit numbers, three-digits, four-digits, decimals, and beyond.

Materials:

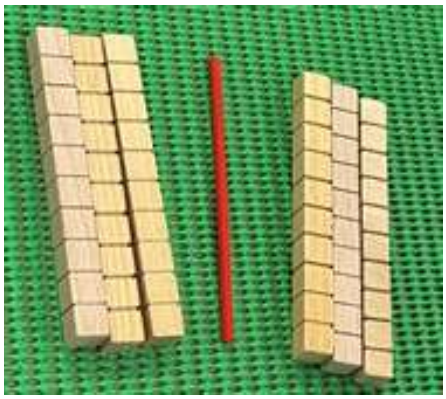
- Tens place value blocks (**wooden MAB** – wooden is critical, as the notches are much clearer than plastic MAB versions).
- Red sticks.
- Grip mats.

Best set-up: Students work individually on grip mats with tens blocks.



1. Roll a 10-sided dice (or 20-sided later) to work out what to multiply by 5.

2. Let's say you roll 6, as in the example here. You are now solving 6 x 5. But first, think 6 x 10 and grab 6 of the tens blocks.



3. 6 x 10 = 60. But you only needed 6 x 5, so halve it. This shows that it is 30. You think tens, then halve to solve for fives.

4. This is similar to the strategy of double one factor, halve the other factor.

5 x 6

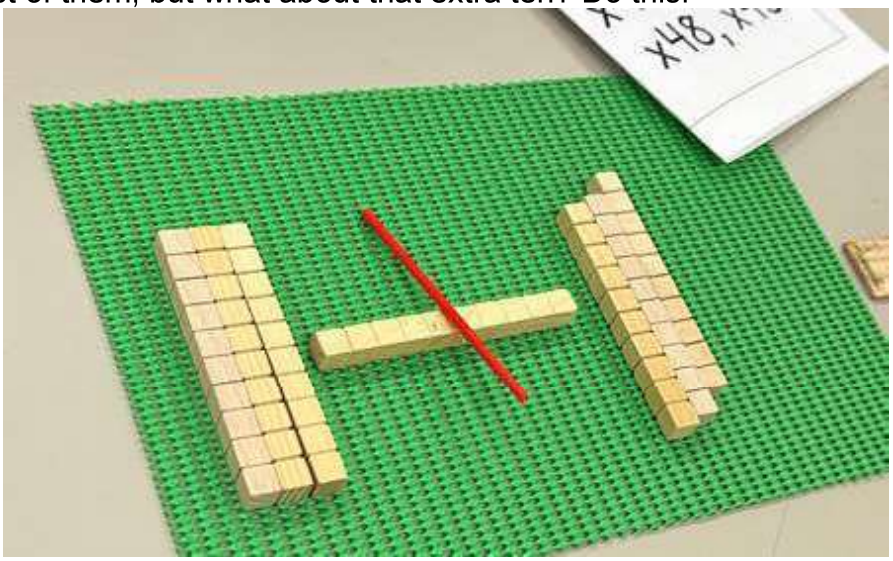
double it halve it

10 x 3

Another example using an odd other number. Let's say you roll 7.

You are now solving 7 x 5. Collect 7 tens, thinking 7 tens is 70.

Now you need to halve it, because you only had 7 fives, not 7 tens. It is easy for most of them, but what about that extra ten? Do this:



Halve the extra ten by placing it horizontal and seeing it is 5 and 5. So you can see the answer visually is 35.

Split strategy: To practice halving 70 without the blocks, try thinking half of 60 (that's 30), half of 10 (that's 5), so 30 + 5 = 35.

	Support: Work on tens with the same materials, rather than fives, if they are still mastering tens for now. If they are beyond the tens, work on nines, as these are generally easier than fives because they only involve taking away one group, rather than halving the entire collection.
	Extension 1: Try the strategy for much larger numbers, such as by rolling three 10-sided dice. For example, 789 x 5. Think 789 x 10, then halve it. Check with a calculator, does the strategy work every time?
	Extension 2: Work out the inverse strategy for division; that is, what is the best way to divide by 5? *Answer for teachers:* Divide by ten, double it, because fives are extremely closely related to the tens. **Misconception alert:** Divide by ten, halve it. You do not halve it, as that would be dividing by 20. You need to double the division by ten, since sharing between 5 people gives you double what you would receive if you were sharing between ten people.
	Extension 3: Inventing related strategies: Next invent other related strategies, such as how to multiply by 20, divide by 20, multiply by 40, divide by 40, multiply by 50 and divide by 50, divide by 500, and so on. **Link to factors:** If you are multiplying by 20, you can break this into 2 x 10 (x10 then double it). Or 4 x 5 (dd then x10 halve it; that is, you can double double, then multiply by 5 by thinking x10 halve it). In this way, once you know the single-digit strategies, most of these can work to solve other multiplications by larger numbers. **Thinking strategies are superior to vertical algorithms:** These are far more applicable in the long-term and a far better mental exercise, in terms of number sense, than a vertical algorithm. The problem with the algorithm is that it provides so much simplification and independent column-based recipes, that it actually lacks depth of thinking, as well as being devoid of whole number place value or strategies that are workable and interesting to try mentally. This is not what we should be aiming for (to reduce the amount of thinking that occurs in mathematics lessons), but algorithms do precisely that. **Estimate first:** Before solving any equation, be sure to estimate first.

